Statistical Language Modeling

From N-grams to Transformers

Gustave Cortal

Laboratoire
Méthodes
Formelles

école normale supérieure paris-saclay

université PARIS-SACLAY

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Tokenization

Tokenization is splitting text into individual words or **tokens**

Multiple challenges:

- Different delimiters: spaces, punctuation
- Contractions: "can't" $\rightarrow$ "can not"
- Special cases: dates, numbers, URLs, hashtags, email addresses

Tokenization

Input

"Natural language processing enables computers to understand human language."

Tokenized output

Natural, language, processing, enables, computers, to, understand, human, language, .

Tokenization

Input

"Dr. Smith's email, dr.smith@example.com, isn't working since 01/02/2023; try reaching out at (555) 123-4567 in San Francisco."

Tokenized output

Dr., Smith's, email, ,, dr.smith@example.com, ,, isn't, working, since, 01/02/2023, ;, try, reaching, out, at, (, 555,), 123-4567, in, San Francisco, .

Tokenization

Rule-based approach

Use predefined rules, like splitting by spaces or punctuation using regular expressions

Machine learning approach

Learn from data to handle complex cases, e.g., using Byte-Pair Encoding subword tokenization

N-grams

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What is a language model?

A **language model** is a probabilistic model that:

- computes the **probability of a sequence of words** S
  $P(S) = P(w_1, w_2, ..., w_n)$
- computes the **probability of an upcoming word**
  $P(w_5 | w_1, w_2, w_3, w_4)$

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For example, for speech recognition:
$P(\text{I saw a van}) >> P(\text{eyes awe of an})$

How to compute P(S)?

Definition of **conditional probabilities**:

$$P(B|A) = P(A, B)/P(A)$$
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Applying the chain rule to compute the joint probability of words in a sentence:

$$P(\text{I am Gustave}) = P(\text{I})P(\text{am}|\text{I})P(\text{Gustave}|\text{I am})$$

How to estimate these probabilities?

Can we just count and divide?

$$P(\text{processing}|\text{I am Gustave, I love natural language}) = \frac{\text{Count(I am Gustave, I love natural language processing)}}{\text{Count(I am Gustave, I love natural language)}}$$

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$\rightarrow$ We'll never see enough data for estimating long sentences

N-grams are Markov models

Markov assumption uses a limited context window to approximate $P(\text{processing}|\text{I am Gustave, I love natural language})$

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$P(\text{processing}|\text{natural language})$ *Trigram*

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$\rightarrow$ Language has long-distance dependencies, therefore n-grams are insufficient models of language

Example: estimating bigram probabilities

Estimation using $P(w_i|w_{i-1}) = \frac{\text{count}(w_i, w_{\mathbf{i-1}})}{\text{count}(w_{\mathbf{i-1}})}$

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<s> I am Gustave </s>
<s> Gustave I am </s>
<s> I love natural language processing </s>

$$P(\text{am}|\text{I}) = \frac{\text{count}(\text{am}, \text{I})}{\text{count}(\text{I})} = \frac{2}{3}$$

How to evaluate performance?

We calculate probabilities on a training set and evaluate on the unseen test set

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We want a language model (LM) that best predicts the test set

Therefore, a good LM assigns a higher probability to the test set than another LM

If the test set has n tokens, then $P(\text{test set}) = P(w_1, w_2, ..., w_n)$

$$P_{\text{good LM}}(\text{test set}) > P_{\text{bad LM}}(\text{test set})$$

Perplexity

Probability depends on the number of tokens, the longer the text, the smaller the probability

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$\rightarrow$ We normalize by the number of tokens to have a metric per token:

$$\text{Perplexity(test set)} = P(w_1, w_2, ..., w_n)^{-\frac{1}{n}}$$

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Minimizing perplexity is the same as maximizing probability

Practical issues

Due to **unknown words**, bigrams with zero probability drop sentence probabilities to zero and prevent us from calculating perplexity

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$\rightarrow$ **Add-1 smoothing** pretends we saw each word one more time than we did

$$P(w_i|w_{i-1}) = \frac{\text{count}(w_i, w_{i-1}) + 1}{\text{count}(w_{i-1}) + V}$$

where V is the vocabulary size

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To avoid **underflow**, every computation is performed in log space

$$\log(p_1 \times p_2 \times p_3) = \log(p_1) + \log(p_2) + \log(p_3)$$

Better n-grams using backoff or interpolation methods

Backing off through progressively shorter context models under certain conditions. For example, use trigram if $\text{count}(w_i, w_{i-1}, w_{i-2}) > 0$, otherwise use bigram.

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Backing off through progressively shorter context models under certain conditions. For example, use trigram if $\text{count}(w_i, w_{i-1}, w_{i-2}) > 0$, otherwise use bigram.

Interpolation methods train individual models for different n-gram orders and then interpolate them together.

$$\hat{P}(w_n|w_{n-2}, w_{n-1}) = \lambda_1 P(w_n|w_{n-2}, w_{n-1}) + \lambda_2 P(w_n|w_{n-1}) + \lambda_3 P(w_n)$$

where $\sum_{i=1}^{3} \lambda_i = 1$

From n-gram to neural network language models

Neural network language models solve major problems with n-grams

- The number of parameters increases exponentially as the n-gram order increases
- N-grams have no way to generalize from training to test set

Neural language models instead **project words into a continuous space** in which words with similar contexts have similar representations

How to represent word meaning?

Word meaning as a point in a multidimensional space

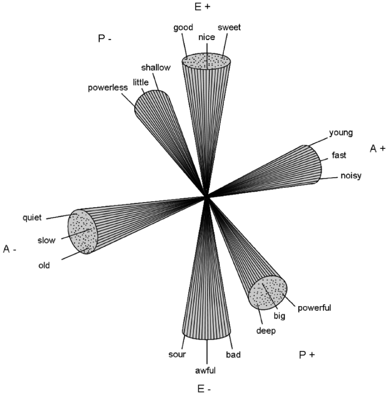


Figure: A three-dimensional affective space of connotative meaning by Osgood et al. (1957)

How to represent word meaning?

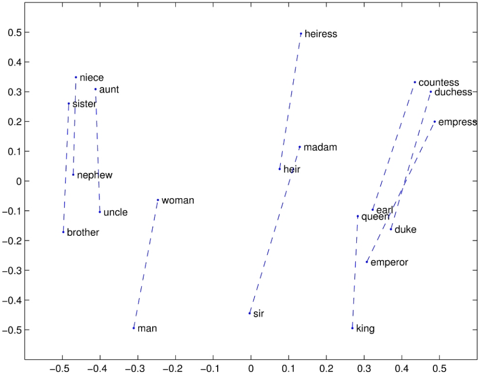
Defining meaning by linguistic distribution

The meaning of a word is its use in a language, Ludwig Wittgenstein (1953)

If A and B have almost identical environments (words around them), then they are synonyms, Zellig Harris (1954)

How to represent word meaning?

Word meaning as a point in a multidimensional space + Defining meaning by linguistic distribution = **Defining meaning as a point in a multidimensional space based on linguistic distribution**



The meaning of a word is a vector called an **embedding**

Components of a machine learning classifier

- A feature representation of the input x
- A classification function that computes the estimated class
- An objective function for learning (e.g., cross-entropy loss)
- An algorithm for optimizing the objective function (e.g., stochastic gradient descent)

Feedforward neural networks

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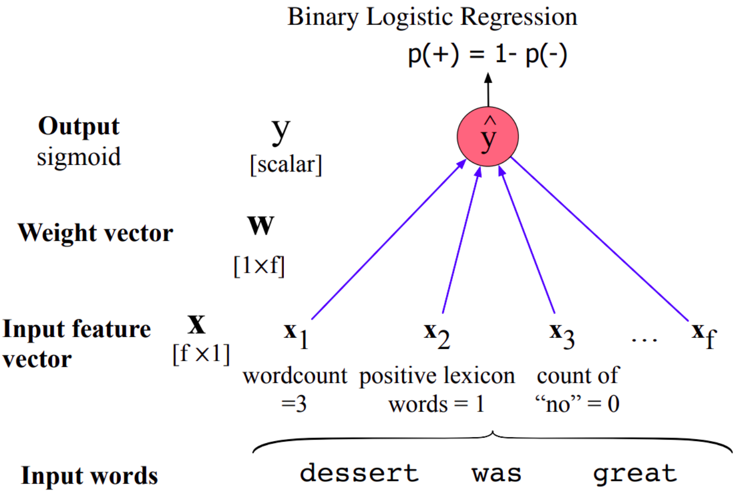
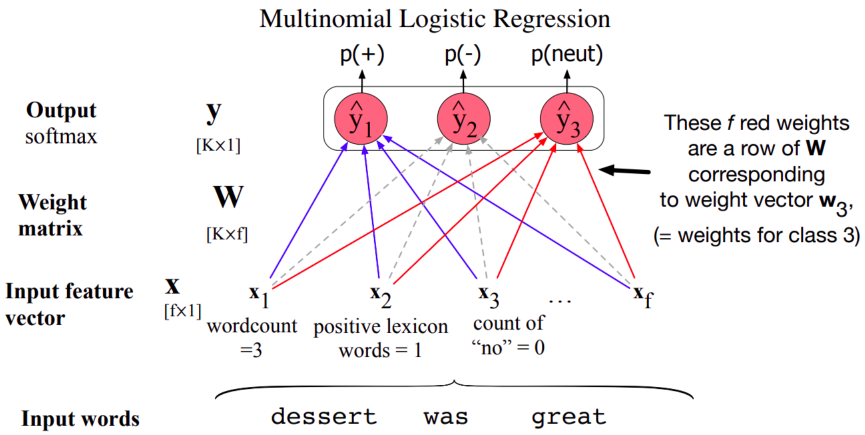
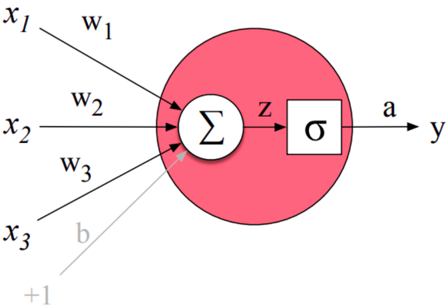
Binary logistic regression



Multinomial logistic regression



Neural unit



$$y = \sigma(w \cdot x + b) = \frac{1}{1 + \exp(-(w \cdot x + b))}$$

Activation functions (1)

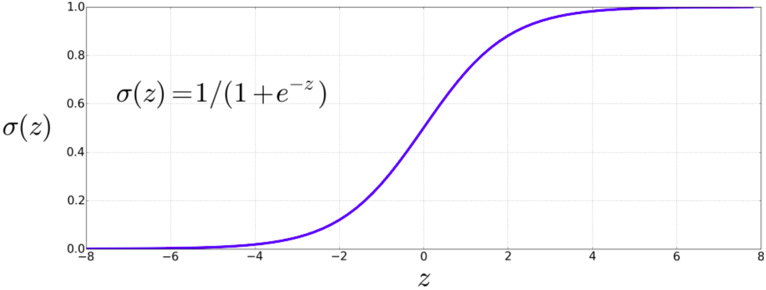


Figure: Sigmoid function.

Activation functions (2)

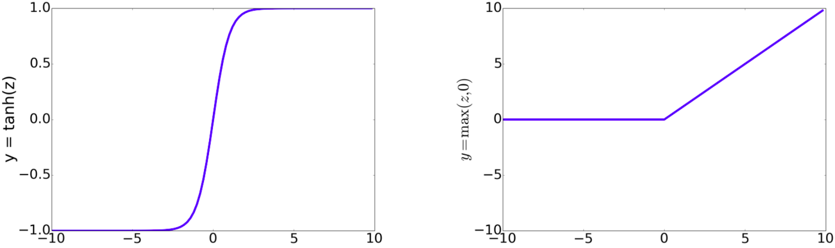
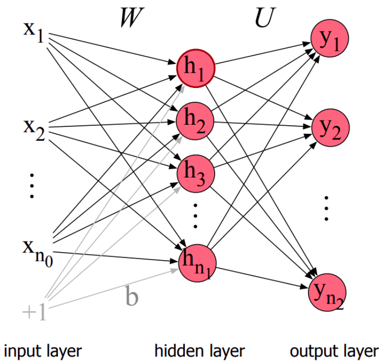


Figure: Tanh and ReLU functions.

$$y = \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$y = \mathrm{ReLU}(z) = \max(z, 0)$$

Feedforward network (1)



$$h = \sigma(Wx + b)$$
$$z = Uh$$
$$y = \mathrm{softmax}(z)$$

Feedforward network (2)

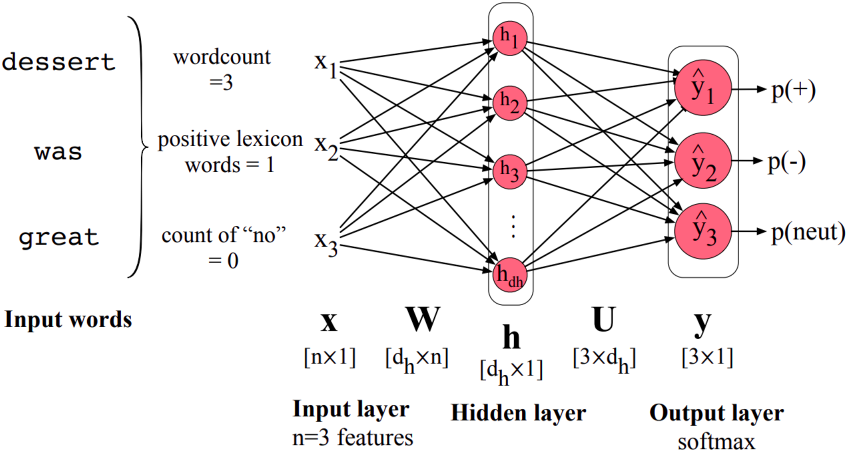


Figure: Feedforward network sentiment analysis using traditional hand-built features.

Feedforward network (3)

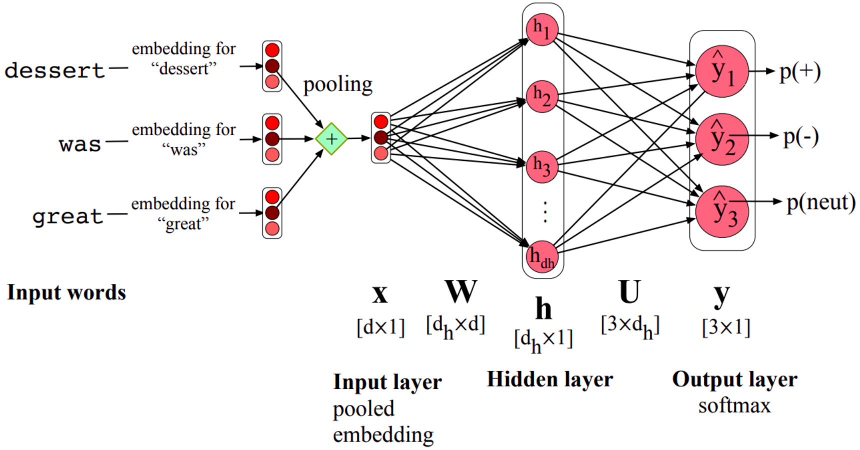


Figure: Feedforward network sentiment analysis using a pooled embedding.

Without activation functions, a multi-layer NN is equivalent to a single-layer NN

Consider the first two layers of a neural network with purely linear transformations:

$$
\begin{aligned}
z^{[1]} &= W^{[1]}x + b^{[1]} \\
z^{[2]} &= W^{[2]}z^{[1]} + b^{[2]}
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$$

The operations performed by the network can be combined and simplified as follows:

$$
\begin{aligned}
z^{[2]} &= W^{[2]}(W^{[1]}x + b^{[1]}) + b^{[2]} \\
&= W^{[2]}W^{[1]}x + W^{[2]}b^{[1]} + b^{[2]} \\
&= W_0 x + b_0
\end{aligned}
$$

Loss function

The loss function for a single example x in the context of a multi-class classification problem, with K output classes, is defined as the cross-entropy loss L_{CE}:

$$L_{CE}(\hat{y}, y) = -\sum_{k=1}^{K} y_k \log(\hat{y}_k)$$

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The loss L_{CE} for a prediction $\hat{y}$ and true label y, focusing on the correct class c, is represented as:

$$\begin{aligned} L_{CE}(\hat{y}, y) &= -\log(\hat{y}_c) \\ &= -\log\left(\frac{\exp(z_c)}{\sum_{j=1}^{K} \exp(z_j)}\right) \end{aligned}$$

Computing the gradients

For deep networks, computing the gradients for each weight is difficult, since we are computing the derivative with respect to weight parameters that appear all the way back in the very early layers of the network

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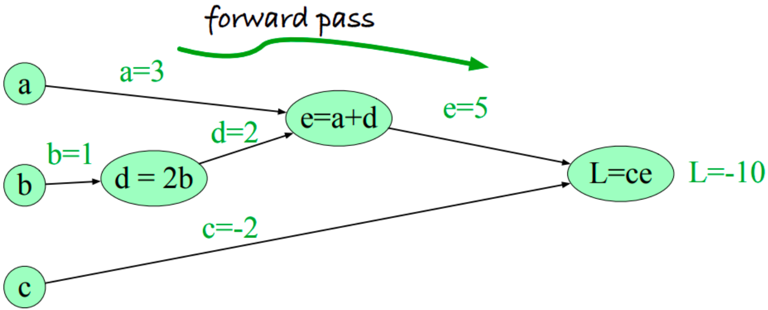
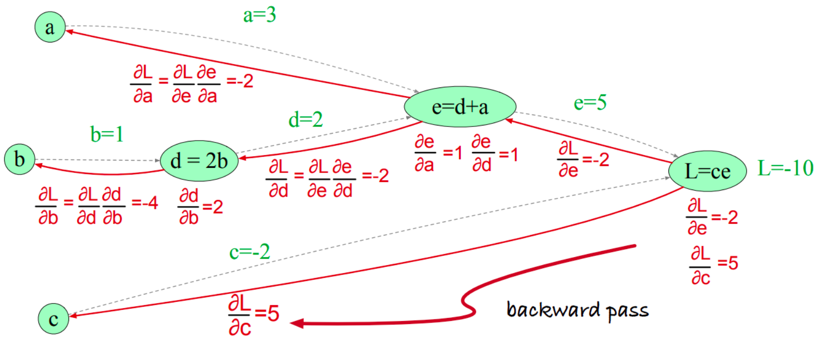
The solution to computing this gradient is an algorithm called **error backpropagation**.

Forward pass



$$L(a, b, c) = c(a + 2b)$$

Backward pass



$$L(a, b, c) = c(a + 2b)$$

Backpropagation calculus, 3Blue1Brown's video

https://www.youtube.com/watch?v=Ilg3gGewQ5U

Feedforward NN for language modeling

A feedforward neural language model takes as input at time t a representation of some number of previous words (w_{t-1}, w_{t-2}, etc.) and outputs a probability distribution over possible next words

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Like the n-gram, it approximates the probability of a word given the entire prior context by approximating based on the $n-1$ previous words:

$$P(w_t|w_1, ..., w_{t-1}) \approx P(w_t|w_{t-N+1}, ..., w_{t-1})$$

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$$P(w_t|w_1, ..., w_{t-1}) \approx P(w_t|w_{t-N+1}, ..., w_{t-1})$$

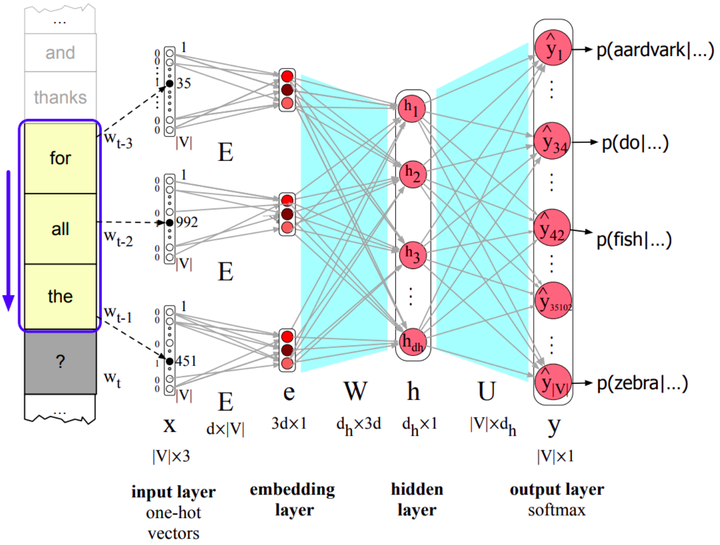
Unlike n-gram models, neural language models can handle much longer histories, generalize better over contexts of similar words, and are more accurate at word prediction.

Feedforward NN for language modeling

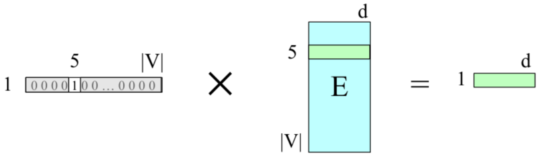
The equations for a neural language model with a window size of 3, given one-hot input vectors for each input context word, are:

$$
\begin{aligned}
e &= [Ex_{t-3}; Ex_{t-2}; Ex_{t-1}] \\
h &= \sigma(We + b) \\
z &= Uh \\
\hat{y} &= \text{softmax}(z)
\end{aligned}
$$

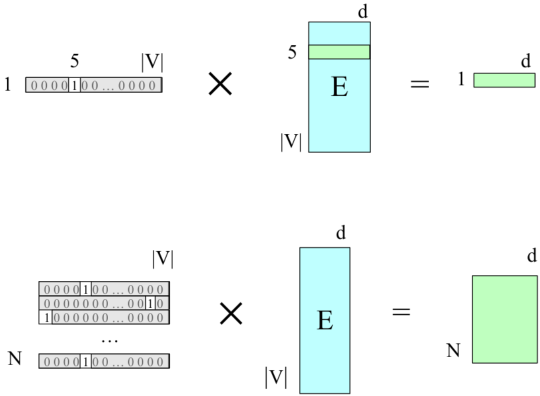
Feedforward NN for language modeling



Embedding



Embedding



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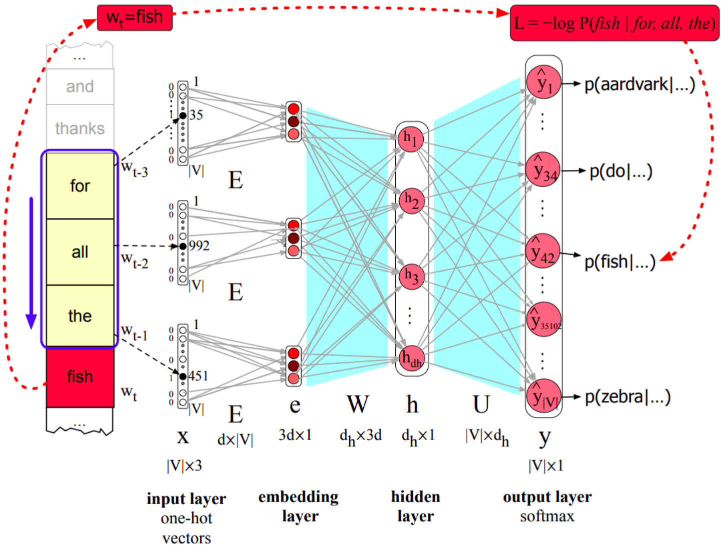
Feedforward NN for language modeling



How to improve the training?

Find good **hyperparameters**: batch size, learning rate, activation functions, number of hidden layers, number of neural units

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Apply **regularization** methods: normalize input values, add dropout, add weight decay

Other techniques include label smoothing, cutting gradients norm, augmenting data, using good weights initialization, gradient descent with momentum (Adam optimizer), etc.

Exercices

Using PyTorch, you need to:

- Implement the logistic regression
- Implement a multi-layer feedforward network for text classification based on word2vec features
- Play with hyperparameters (see slide "How to improve the training?")
- Implement a multi-layer feedforward network for language modeling (optional)
- Study the skip-gram model (optional, see notebook from the last course)

The goal is to have a workable PyTorch training loop for your project!

Recurrent neural networks and attention mechanisms

Gustave Cortal

Introduction

Language is a **temporal** phenomenon

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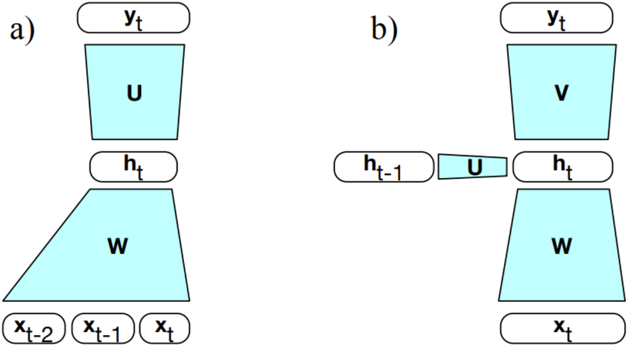
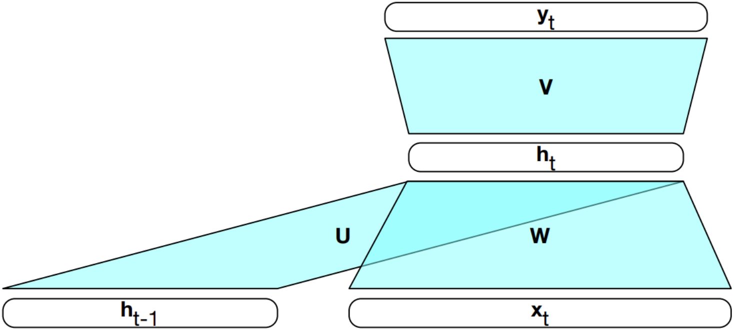
Feedforward neural networks assumed **simultaneous access**: for language modeling, they look only at a fixed-size window of words, then slide this window over the input

Recurrent neural networks handle the temporal nature of language without using arbitrary fixed-sized windows: the hidden layer from the previous step provides a **memory** that encodes earlier processing and informs the decisions to be made at later steps

Feedforward vs recurrent neural networks

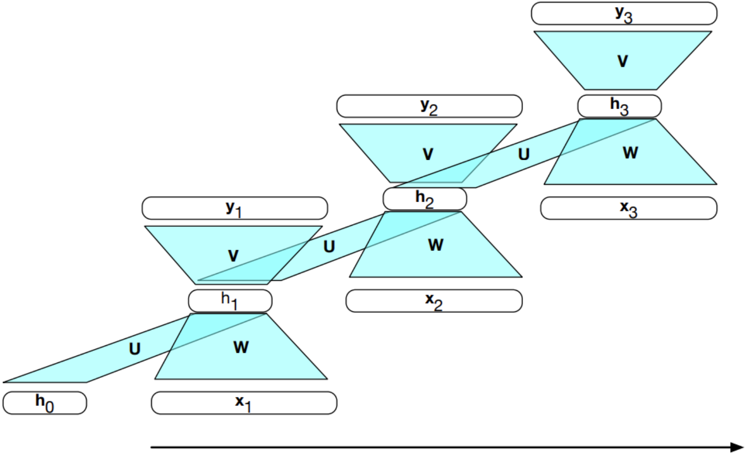


Recurrent neural networks

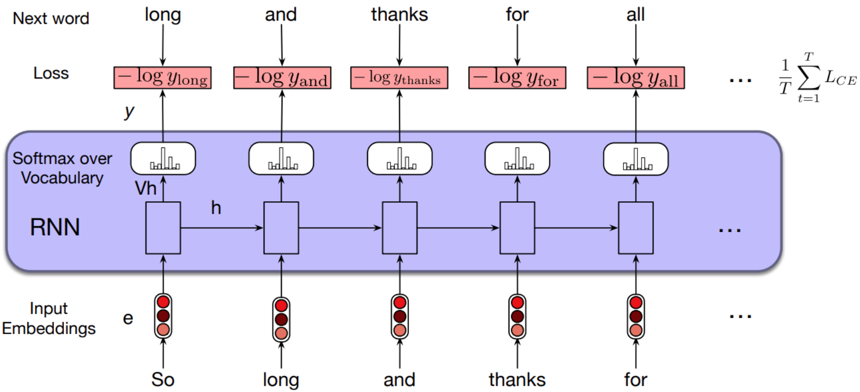


$$h_t = g(Uh_{t-1} + Wx_t)$$
$$y_t = softmax(Vh_t)$$

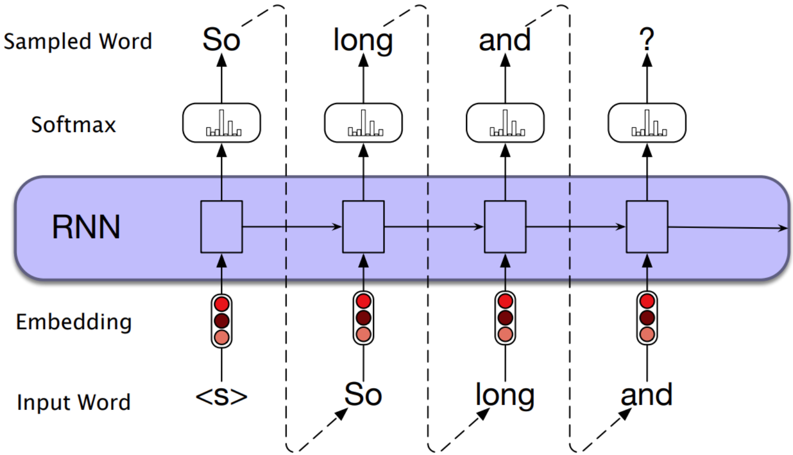
Recurrent neural networks



RNNs for language modeling



Sampling



Sampled Word
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and
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Softmax
RNN
Embedding
Input Word
<s>
So
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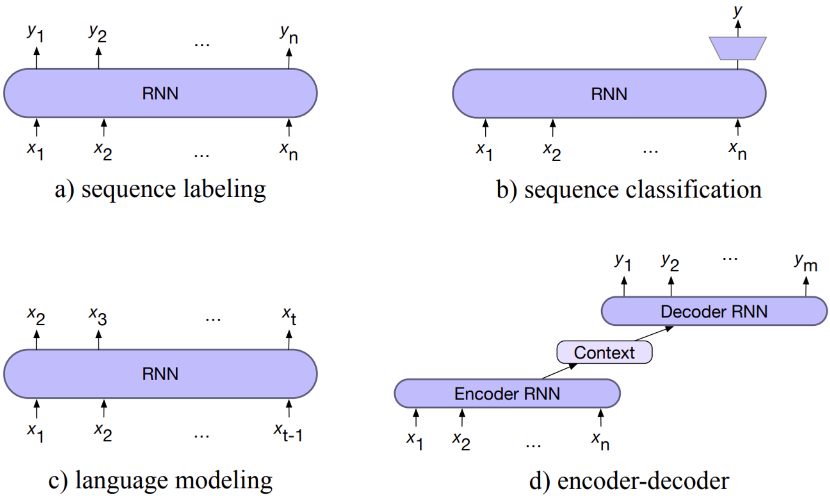
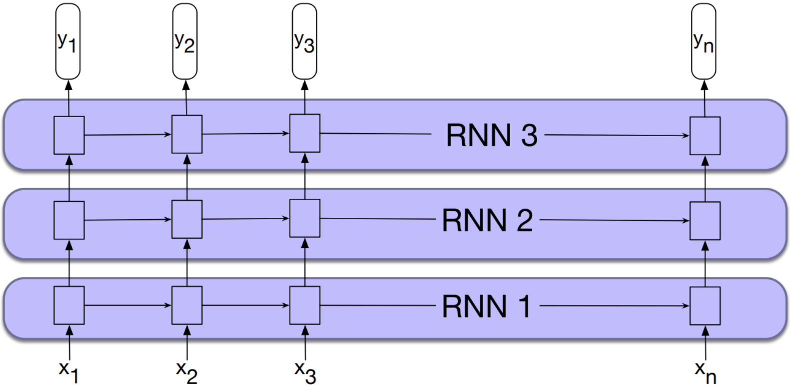
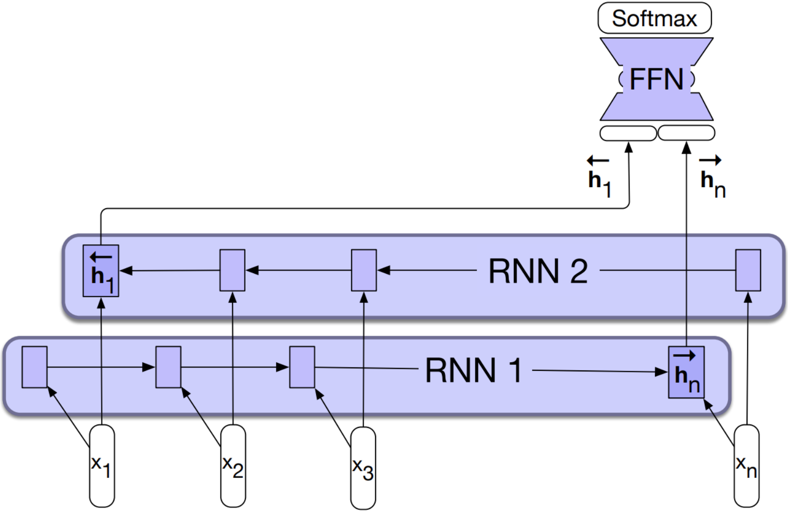
RNNs for other tasks



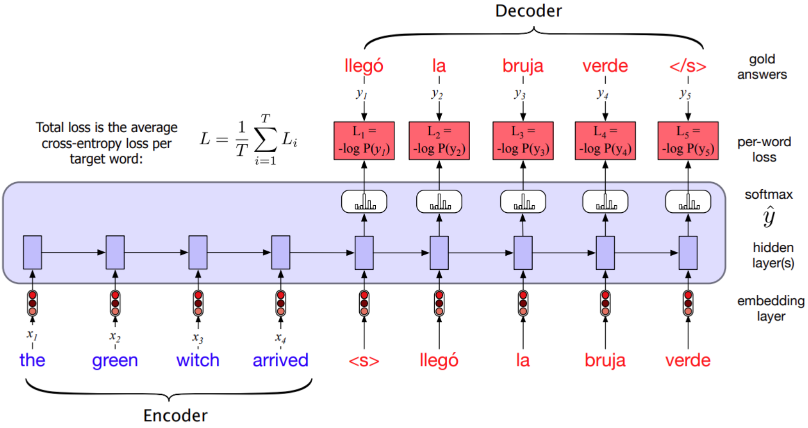
Stacked RNNs



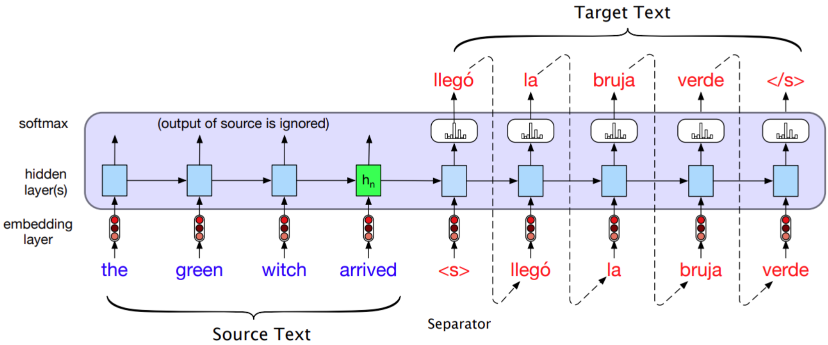
Bidirectional RNNs



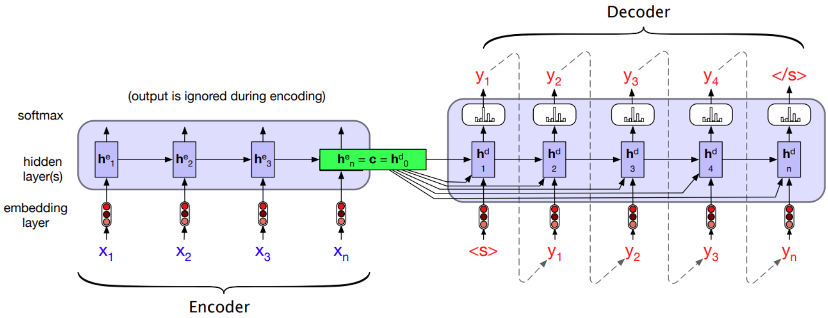
Training with encoder-decoder networks



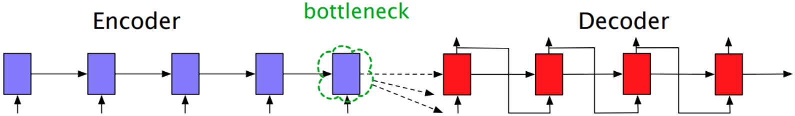
Inference with encoder-decoder networks



Final hidden state as a fixed context vector for the decoder

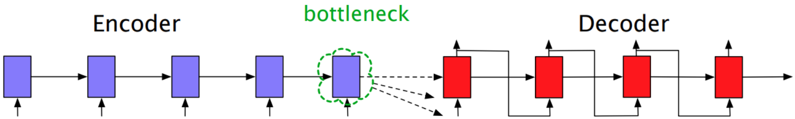


The final hidden state acts as a bottleneck



This final hidden state must represent everything about the meaning of the source text

The final hidden state acts as a bottleneck



This final hidden state must represent everything about the meaning of the source text

However, information at the beginning of the sentence may not be equally well represented in the context vector

Attention mechanisms: introduction

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Attention thus replaces the static context vector with one that is **dynamically** derived from the encoder hidden states, **different** for each token in

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Then, apply a softmax to create a **vector of weights**, α_{ij}, that tells the proportional relevance of each encoder hidden state j to the prior hidden decoder state, h^d_{i-1}:

$$\alpha_{ij} = \frac{\exp(\text{score}(h^d_{i-1}, h^e_j))}{\sum_k \exp(\text{score}(h^d_{i-1}, h^e_k))}$$

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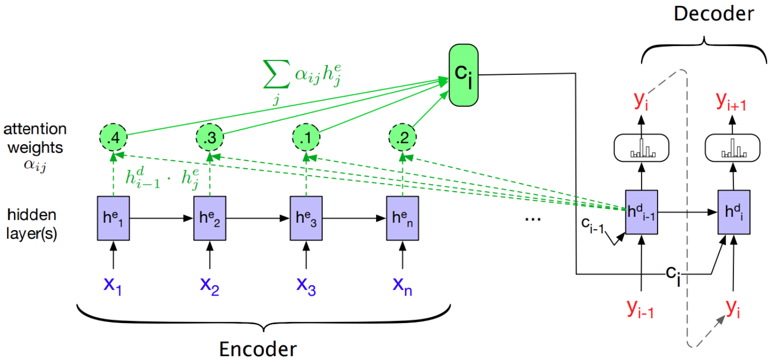
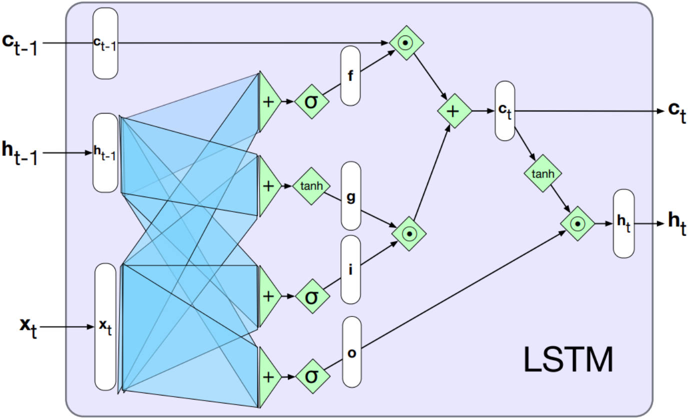
Finally, compute a fixed-length context vector for the current decoder state by taking a weighted average over all the encoder hidden states:

$$c_i = \sum_j \alpha_{ij} h^e_j$$

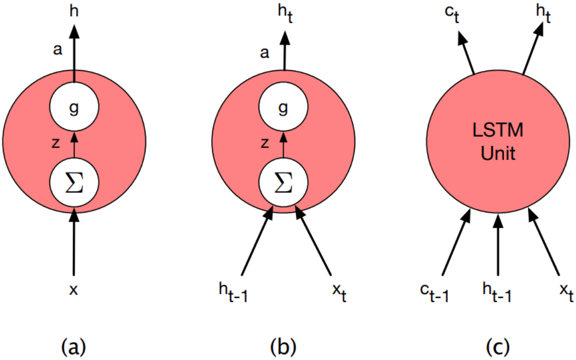
Encoder-decoder networks with dot-product attention



Long Short Term Memory network



FNN vs RNN vs LSTM units



Transformers

Gustave Cortal

Transformers *vs* recurrent neural networks

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Transformers *vs* recurrent neural networks

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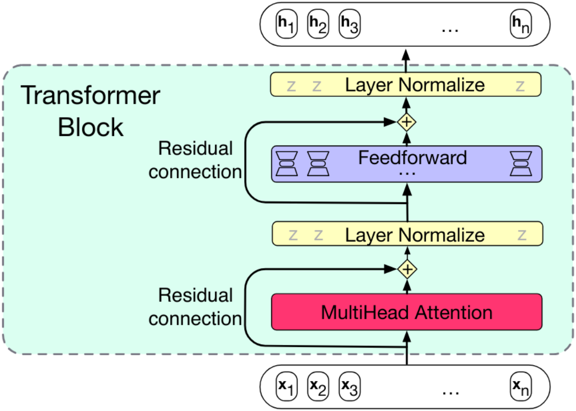
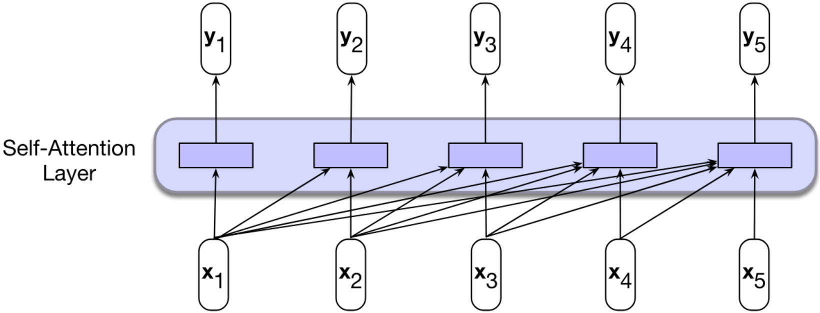
Unlike RNNs, the computations at each time step are **independent of all the other steps** and, therefore, can **be performed in parallel**

Transformer block



Self-attention layer



Self-attention directly extracts and uses information from arbitrarily large contexts without passing it through intermediate recurrent connections

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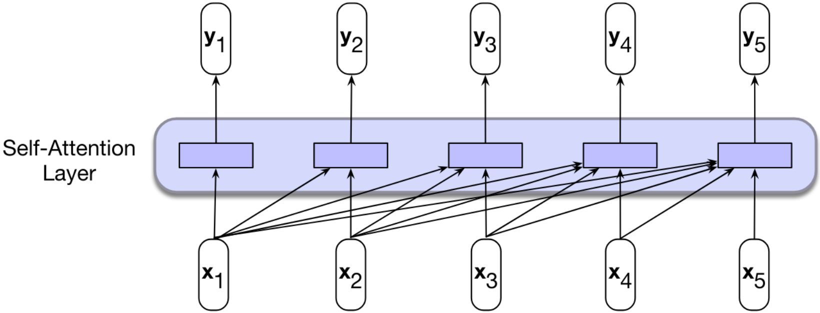

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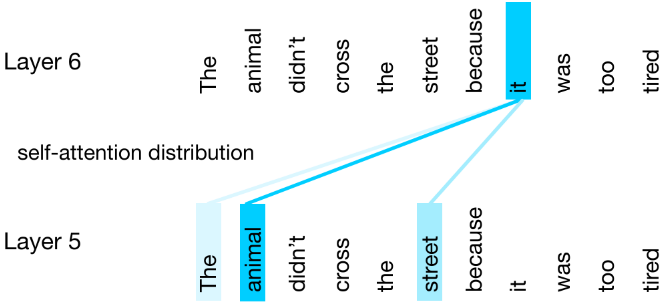
Attention visualization



Main idea of attention mechanisms

An attention-based approach is a set of **comparisons to relevant items** in some context, a **normalization** of those scores to provide a probability distribution, and a **weighted sum** using this distribution

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$$\begin{aligned}\alpha_{ij} &= \text{softmax}(\text{score}(x_i, x_j)) \quad \forall j \leq i \\ &= \frac{\exp(\text{score}(x_i, x_j))}{\sum_{k=1}^{i} \exp(\text{score}(x_i, x_k))} \quad \forall j \leq i\end{aligned}$$

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Finally, we generate an output value y_i by taking the **sum** of the inputs seen so far, **weighted** by their respective α value.

$$y_i = \sum_{j \leq i} \alpha_{ij} x_j$$

Attention with queries, keys and values

But transformers create a **more sophisticated way** of representing how words can contribute to the representation of longer inputs. Consider the three roles each input embedding plays during the attention process:

- As the current focus of attention when being compared to all of the other preceding inputs $\rightarrow$ **query**
- In its role as a preceding input being compared to the current focus of attention $\rightarrow$ **key**
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To capture these three different roles, transformers introduce weight matrices W_Q, W_K, and W_V. These weights project each input vector x_i into a representation of its role as a key, query, or value:

$$q_i = W_Q x_i,$$
$$k_i = W_K x_i,$$
$$v_i = W_V x_i$$

$x_i \in \mathbb{R}^{d \times 1}$, $W_Q \in \mathbb{R}^{d \times d}$, $W_K \in \mathbb{R}^{d \times d}$, and $W_V \in \mathbb{R}^{d \times d}$.

Attention with queries, keys and values

Given these projections, the score between a current focus of attention, x_i, and an element in the preceding context, x_j, consists of a dot product between its query vector q_i and the preceding element's key vectors k_j:

$$\text{score}(x_i, x_j) = q_i \cdot k_j$$

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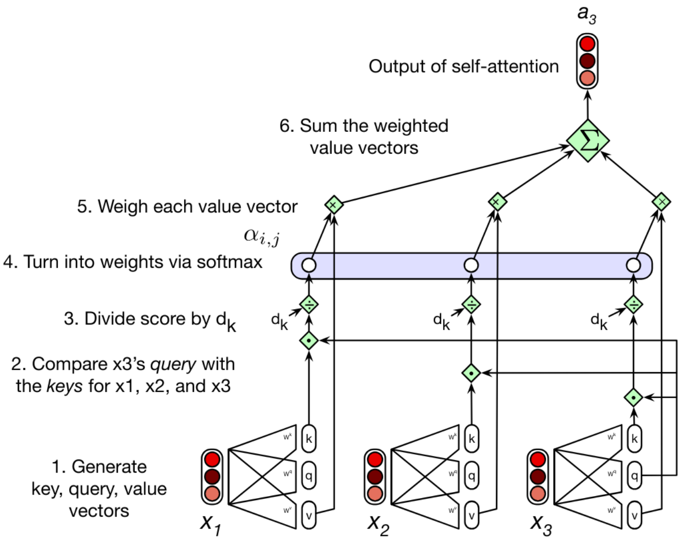
Exponentiating large values can lead to numerical issues. To avoid this, we **scale** the dot-product by a factor related to the size of the embeddings:

$$\text{score}(x_i, x_j) = \frac{q_i \cdot k_j}{\sqrt{d}}$$

Attention with queries, keys and values



Parallelization

Since each output y_i is computed independently, the entire process can be parallelized by taking advantage of matrix multiplication

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Input tokens are packed into a single matrix $X \in \mathbb{R}^{N \times d}$. We multiply X by the key, query, and value matrices:

$$Q = XW_Q; \quad K = XW_K; \quad V = XW_V$$

$Q \in \mathbb{R}^{N \times d}$, $K \in \mathbb{R}^{N \times d}$, and $V \in \mathbb{R}^{N \times d}$

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We've reduced the self-attention step for a sequence of N tokens:

$$\text{SelfAttention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d}}\right) V$$

Masked attention matrix

	q1•k1	−∞	−∞	−∞	−∞
	q2•k1	q2•k2	−∞	−∞	−∞
N	q3•k1	q3•k2	q3•k3	−∞	−∞
	q4•k1	q4•k2	q4•k3	q4•k4	−∞
	q5•k1	q5•k2	q5•k3	q5•k4	q5•k5

N

QK^T results in a score for each query value to every key value, including those that follow the query

Masked attention matrix

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q1•k1	$-\infty$	$-\infty$	$-\infty$	$-\infty$
q2•k1	q2•k2	$-\infty$	$-\infty$	$-\infty$
q3•k1	q3•k2	q3•k3	$-\infty$	$-\infty$
q4•k1	q4•k2	q4•k3	q4•k4	$-\infty$
q5•k1	q5•k2	q5•k3	q5•k4	q5•k5

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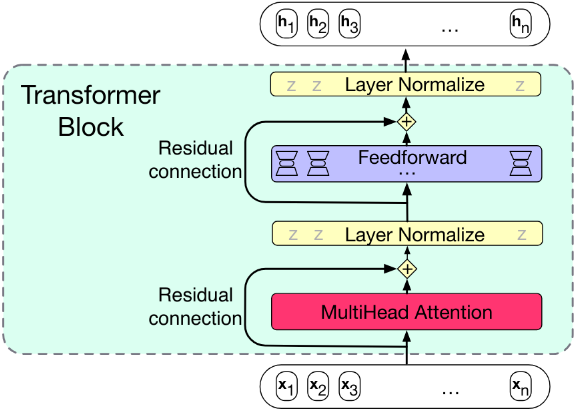
QK^T results in a score for each query value to every key value, including those that follow the query

This is inappropriate in language modeling since guessing the next word is pretty simple if you already know it. To fix this, the elements in the upper-triangular portion of the matrix are set to $-\infty$

Transformer block



Multihead attention

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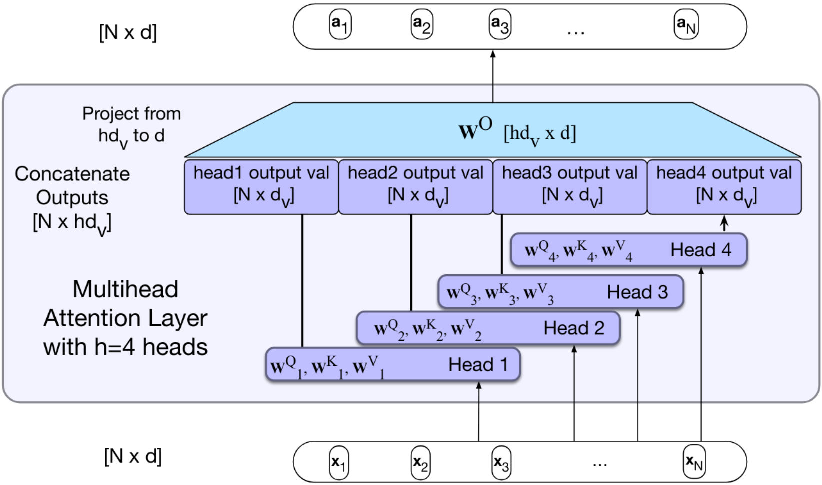
Given these distinct sets of parameters, each head can learn different aspects of the relationships among inputs at the same level of abstraction

Multihead attention



Multihead attention

To implement this notion, each head, i, in a self-attention layer is provided with its own set of key, query, and value matrices: W_i^K, W_i^Q, and W_i^V

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$$\text{MultiHeadAttention}(X) = (\text{head}_1 \oplus \text{head}_2 \ldots \oplus \text{head}_h)W^O$$

$$Q_i = XW_i^Q; \quad K_i = XW_i^K; \quad V_i = XW_i^V$$

$$\text{head}_i = \text{SelfAttention}(Q_i, K_i, V_i)$$

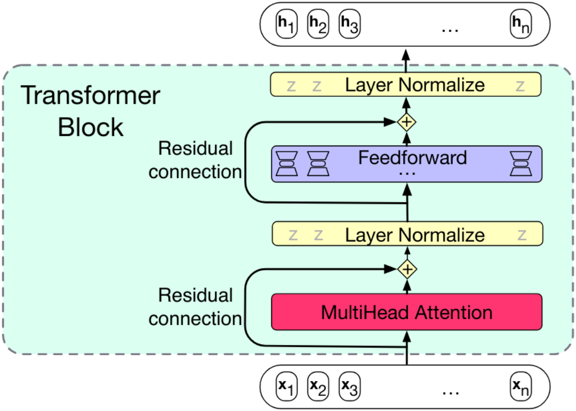
$X \in \mathbb{R}^{N \times d}$

$W_i^Q \in \mathbb{R}^{d \times d_k}$, $W_i^K \in \mathbb{R}^{d \times d_k}$, and $W_i^V \in \mathbb{R}^{d \times d_v}$

$Q \in \mathbb{R}^{N \times d_k}$, $K \in \mathbb{R}^{N \times d_k}$, and $V \in \mathbb{R}^{N \times d_v}$

$W^O \in \mathbb{R}^{hd_v \times d}$

Transformer block



Residual connections

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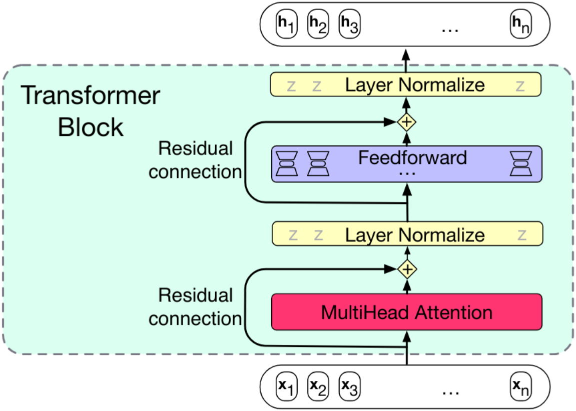
If we think of a layer as one long vector of units, the resulting function computed in a transformer block can be expressed as:

$$
\begin{aligned}
O &= \mathrm{LayerNorm}(\mathbf{X} + \mathrm{SelfAttention}(X)) \\
H &= \mathrm{LayerNorm}(\mathbf{O} + \mathrm{FFN}(O))
\end{aligned}
$$

Transformer block



Layer normalization

$$O = \textbf{LayerNorm}(X + \text{SelfAttention}(X))$$
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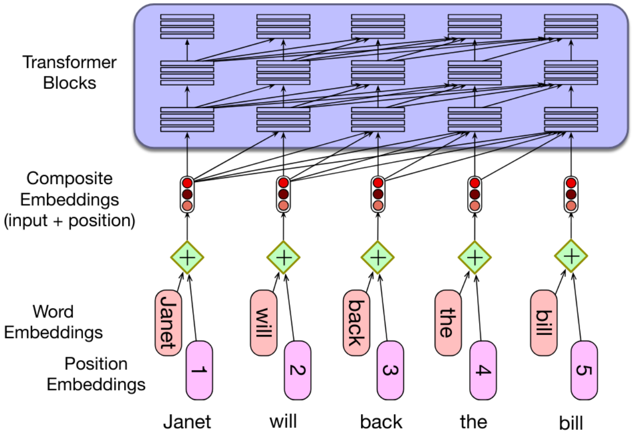
We calculate the mean, μ, and standard deviation, σ, over the elements of the vector to be normalized. Given a hidden layer with dimensionality d, these values are calculated as follows:

$$\mu = \frac{1}{d}\sum_{i=1}^{d} x_i$$

$$\sigma = \sqrt{\frac{1}{d}\sum_{i=1}^{d}(x_i - \mu)^2}$$

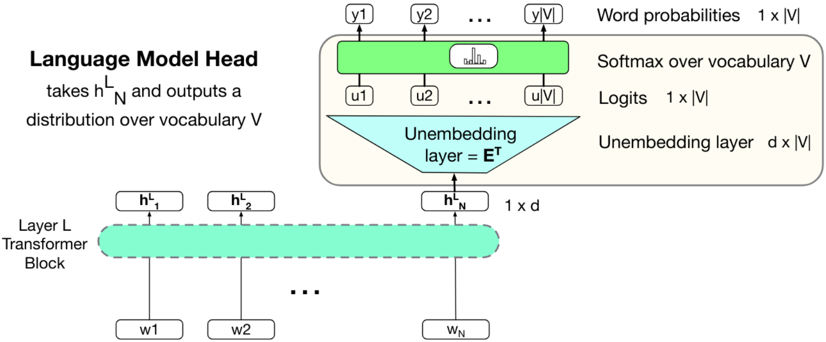
$$\hat{x} = \frac{(x - \mu)}{\sigma}$$

Positional encoding

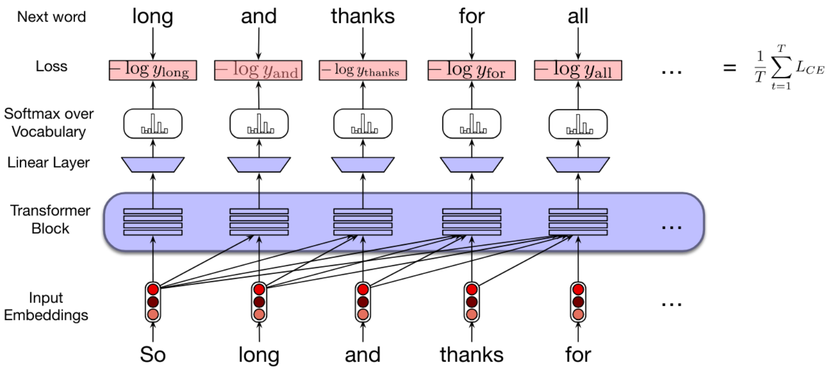


Train positional embeddings or use a static function that maps integer inputs to real-values vectors

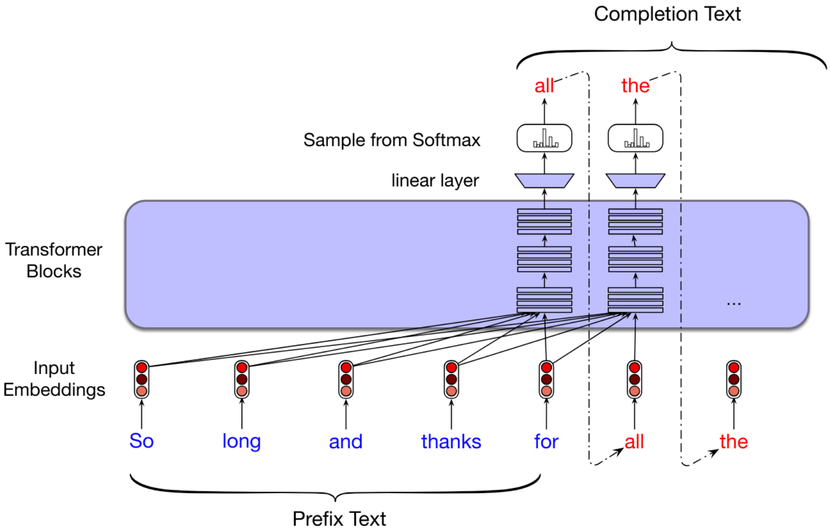
Language model head



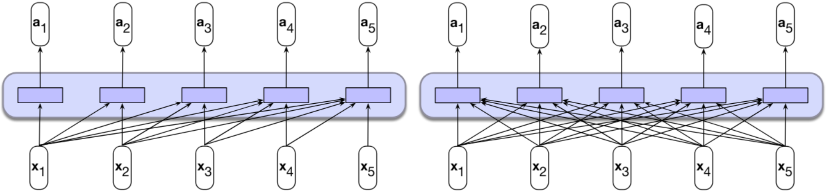
Language modeling using next word prediction



Conditional generation



Causal *vs* bidirectional language model



a) A causal self-attention layer

b) A bidirectional self-attention layer

Attention matrix for bidirectional language model

N

q1•k1	q1•k2	q1•k3	q1•k4	q1•k5
q2•k1	q2•k2	q2•k3	q2•k4	q2•k5
q3•k1	q3•k2	q3•k3	q3•k4	q3•k5
q4•k1	q4•k2	q4•k3	q4•k4	q4•k5
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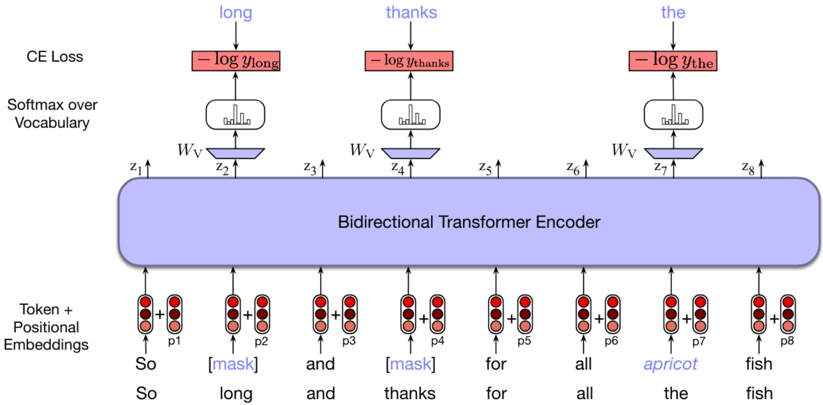
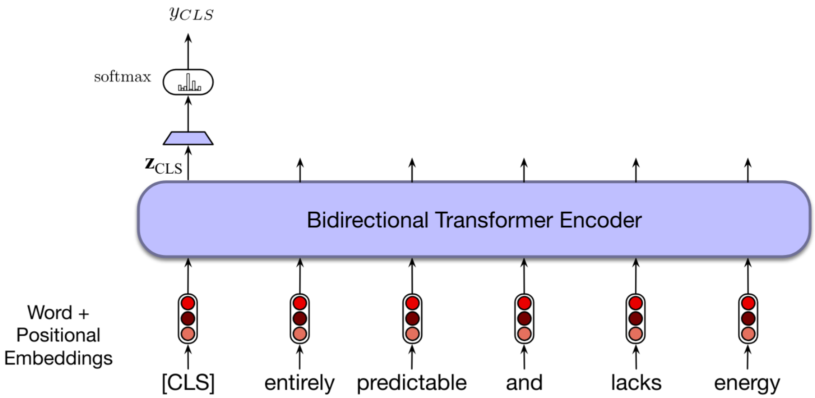
N

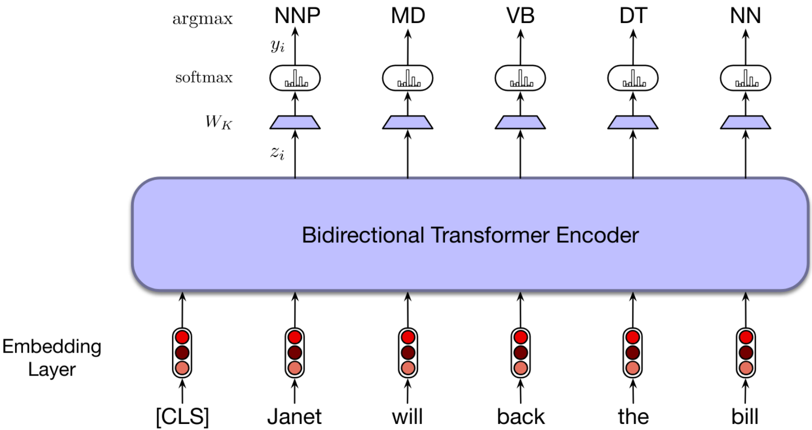
Masked language modeling



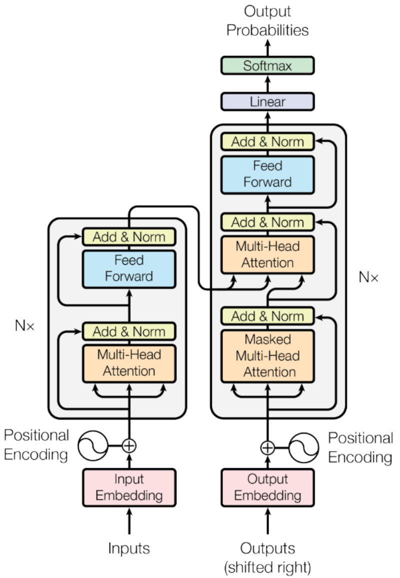
Sequence classification



Token classification



Transformer architecture from *Attention is All you Need*



Architecture, size, and hyperparameters of GPT-3 from *Language Models are Few-Shot Learners*

Model Name	n_{params}	n_{layers}	d_{model}	n_{heads}	d_{head}	Batch Size	Learning Rate
GPT-3 Small	125M	12	768	12	64	0.5M	6.0×10^{-4}
GPT-3 Medium	350M	24	1024	16	64	0.5M	3.0×10^{-4}
GPT-3 Large	760M	24	1536	16	96	0.5M	2.5×10^{-4}
GPT-3 XL	1.3B	24	2048	24	128	1M	2.0×10^{-4}
GPT-3 2.7B	2.7B	32	2560	32	80	1M	1.6×10^{-4}
GPT-3 6.7B	6.7B	32	4096	32	128	2M	1.2×10^{-4}
GPT-3 13B	13.0B	40	5140	40	128	2M	1.0×10^{-4}
GPT-3 175B or "GPT-3"	175.0B	96	12288	96	128	3.2M	0.6×10^{-4}

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Feedforward neural networks handle longer inputs and generalize better compared to N-grams thanks to embeddings, have fixed context windows

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Information flow is better in gated recurrent networks due to better context management

Attention mechanisms solve the bottleneck problem to produce dynamically derived context vectors

Transformers use self-attention layers combined with feedforward layers to handle more complex distant relationships between tokens, enable parallelization due to independent computation between tokens, have fixed context windows

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