The Barwise-Seligman Model of Representation Systems: A Philosophical Explication

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Abstract. As an application of their channel theory, Barwise & Seligman sketched a set-theoretic model of representation systems. Their model has the attraction of capturing many important logical properties of diagrams, but few attempts have been made to apply it to actual diagrammatic systems. We attribute this to a lack of precision in their explanation of what their model is about—what a "representation system" is. In this paper, we propose a concept of representation system on the basis of Barwise & Seligman's original ideas, supplemented by Millikan's theory of reproduction. On this conception, a representation system is a family of individual representational acts formed through a repetitive reproduction process that preserves a set of syntactic and semantic constraints. We will show that this concept lets us identify a piece of reality that the Barwise-Seligman model is concerned with, making the model ready for use in the logical analysis of real-world representation systems.

Keywords: representation systems, diagrams, channel theory.

1 Introduction

Channel theory is an attempt to characterize information flows in our environment from a logical point of view. In their book that develops this theory, Barwise and Seligman outlined a general model of representation systems as one of the theory's principal applications [1, Chapter 20]. This model, which we will call "the B&S model," proposes a general framework in which we can investigate logical-semantical properties of a wide range of representation systems, including the systems of spoken language, written language, physical models, and, most importantly for our purpose, diagrammatic representations.

The B&S model is an abstract model of representation systems, and as such it can be used to describe *classes* of representation systems by characterizing abstract properties common to their members. Results proven in the B&S model concerning representation systems with particular properties will apply to all such representation systems, and consequently, verifying that a new representation system has the desired property can be achieved simply by matching the

T. Dwyer et al. (Eds.): Diagrams 2014, LNAI 8578, pp. 231–245, 2014.

system to the model. Barwise and Seligman have shown, for instance, that their model lets us formally characterize some of the fundamental properties of diagrammatic systems, such as free rides, over-specificity, and auto-consistency, that have direct implications on their cognitive efficacy [2,3]. Thus, the B&S model has significant potential as a formal framework for logical study of diagrammatic representations, and when fully developed, will complement the proof-theoretic and model-theoretic framework that have been productively applied to diagrammatic systems [4,5,6, for example].

To our knowledge, however, few attempts have been made to apply the B&S model to actual diagrammatic systems, not even to reveal those fundamental properties it is known to handle well. The mathematical foundation for the B&S model is laid out explicitly by Barwise and Seligman throughout their book, with all its main components derived from standard set theory. We believe that the model has failed to obtain traction, not because of vagueness in the explication of the model, but rather in the lack of specificity concerning the question of how that model is supposed to fit into reality. What aspects of reality are the individual components of the model supposed to capture? Why are those particular mathematical structures required for that purpose? As to these conceptual issues, Barwise and Seligman give only very general clues, leaving some important questions unanswered. In this paper, we lay a conceptual foundation for the avenue of logical study of diagrams that Barwise and Seligman have pointed to.

The first two thirds of this paper are devoted to these specific mathematical questions. After presenting an overview of the structure of the B&S model (Section 2), we will explain the aspects of the B&S model that are relatively easy to interpret (Section 3). We then attack the parts that are less straightforward—the part that develops a rather unusual "two-tier" semantic theory (Section 4).

As the details are filled in, however, it becomes clear that a most fundamental question is yet to be answered. That is, what the B&S model is a model of. Well, it is a model of representation systems, but what is a representation system anyway? Not knowing what the model is about implies not being confident about what the model applies to, and this presents a block to applying the model.

Drawing on the rather scarce clues provided by Barwise and Seligman, we will reconstruct the notion of a representation system that the B&S model apparently presupposes (Section 5). We propose to understand Barwise and Seligman's notion of representation system on the basis of Millikan's theory of reproduction [7,8]. According to this view, a representation system is a family of individual representational acts formed through repetitive reproduction of new representational acts from temporally preceding representational acts. The syntactic and semantic rules associated with a representation system are then explained as the constraints that these individual acts inherit over the reproduction process.

2 The Structure of the B&S Model

In this section, we will present the B&S model in its bare structure. Our purpose here is not to illustrate or explain the model, but to clearly present the mathematical structure posited in the model as preparation for subsequent exposition.

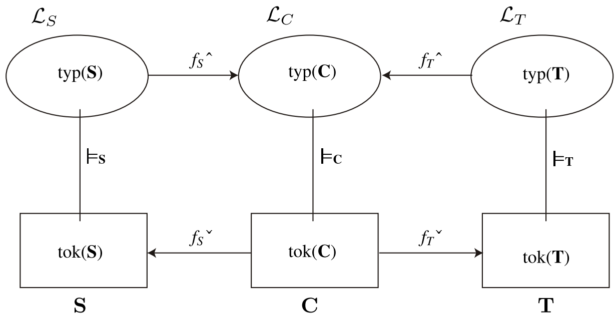

Fig. 1. The Barwise-Seligman Model of Representation Systems

Figure 1 shows the general structure of the model. It is composed of the three *classifications*, three *local logics* on them, and two *infomorphisms* connecting the three classifications. Let us define each component more exactly.

Definition 1 (Classification). *A classification* $\mathbf{A} = \langle tok(\mathbf{A}), typ(\mathbf{A}), \models_{\mathbf{A}}\rangle$ *consists of*

1. *a set, tok*($\mathbf{A}$)*, of objects to be classified, called the* tokens *of* $\mathbf{A}$*,*
2. *a set, typ*($\mathbf{A}$)*, of objects to classify the tokens, called the* types *of* $\mathbf{A}$*, and*
3. *a binary relation,* $\models_{\mathbf{A}}$*, between tok*($\mathbf{A}$) *and typ*($\mathbf{A}$)*.*

Thus, the three classifications involved in the structure in Figure 1 are:

– $\mathbf{S} = \langle \mathrm{tok}(\mathbf{S}), \mathrm{typ}(\mathbf{S}), \models_{\mathbf{S}}\rangle$ (depicted in left),
– $\mathbf{C} = \langle \mathrm{tok}(\mathbf{C}), \mathrm{typ}(\mathbf{C}), \models_{\mathbf{C}}\rangle$ (center), and
– $\mathbf{T} = \langle \mathrm{tok}(\mathbf{T}), \mathrm{typ}(\mathbf{T}), \models_{\mathbf{T}}\rangle$ (right).

We use lowercase Greek letters to refer to types of a classification, and uppercase Greek letters to refer to sets of types. When $a \models_{\mathbf{A}} \alpha$, we say "$a$ is of type α," "a supports α," or "α holds of a."

Given any classification $\mathbf{A}$, we often talk about a pair $\langle \Gamma, \Delta\rangle$ of subsets of typ($\mathbf{A}$). We call such a pair a *sequent* in $\mathbf{A}$. A token a in tok($\mathbf{A}$) is said to *satisfy* a sequent $\langle \Gamma, \Delta\rangle$ if a supports some member of Δ provided a supports all members of Γ. Thus, we are reading Γ conjunctively and Δ disjunctively when we talk about the satisfaction of a sequent.

We use a (subscripted) turnstile $\vdash$ to denote a set of sequents, and write $\Gamma \vdash \Delta$ to mean that $\langle \Gamma, \Delta\rangle$ belongs to $\vdash$. In such a context, we adopt a common abuse of notation for Gentzen sequents. In particular, we omit braces in denoting a unit set (e.g., "$\gamma \vdash \delta$" instead of "$\{\gamma\} \vdash \{\delta\}$") and use a comma to denote the union of sets (e.g., "$\Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2$" instead of "$\Gamma_1 \cup \Gamma_2 \vdash \Delta_1 \cup \Delta_2$").

With this preparation, local logics can be defined in the following way:

Definition 2 (Local Logic). *A* local logic $\mathcal{L} = \langle \mathbf{A}, \vdash_{\mathcal{L}}, N_{\mathcal{L}}\rangle$ *on a classification* $\mathbf{A}$ *consists of*

1. *a set* $\vdash_{\mathcal{L}}$ *of sequents in* $\mathbf{A}$ *satisfying the following closure conditions:*
   Identity *:* $\alpha \vdash_{\mathcal{L}} \alpha$ *for every* $\alpha \in typ(\mathbf{A})$*,*
   Weakening *: If* $\Gamma \vdash_{\mathcal{L}} \Delta$*, then* $\Gamma, \Sigma_1 \vdash_{\mathcal{L}} \Delta, \Sigma_2$ *for any* $\Sigma_1, \Sigma_2 \subseteq typ(\mathbf{A})$*,*
   Global Cut *: If there is a set* $\Sigma \subseteq typ(\mathbf{A})$ *such that* $\Sigma_1, \Gamma \vdash_{\mathcal{L}} \Delta, \Sigma_2$ *for each partition* $\langle \Sigma_1, \Sigma_2 \rangle$ *of* Σ*, then* $\Gamma \vdash_{\mathcal{L}} \Delta$*.*[1]
2. *a subset* $N_{\mathcal{L}} \subseteq tok(\mathbf{A})$*, called the* normal tokens *of* $\mathcal{L}$*, which satisfy all the sequents of* $\vdash_{\mathcal{L}}$*.*

A local logic $\mathcal{L} = \langle \mathbf{A}, \vdash_{\mathcal{L}}, N_{\mathcal{L}} \rangle$ is designed to specify a system of *constraints* governing the classification $\mathbf{A}$. It does the job by specifying a set $\vdash_{\mathcal{L}}$ of sequents in $\mathbf{A}$ that all normal tokens in tok($\mathbf{A}$) are supposed to satisfy. Just what token is normal or abnormal is specified by $N_{\mathcal{L}}$, which carves out the members of tok($\mathbf{A}$) that are normal as far as the local logic $\mathcal{L}$ is concerned.[2] We call a sequent in $\vdash_{\mathcal{L}}$ a *constraint in* the local logic $\mathcal{L}$.

The closure conditions in clause 1 of Definition 2 are required if we are to be able to read the sequents in $\vdash_{\mathcal{L}}$ as the constraints satisfied by any set of tokens whatsoever. For example, if the sequent $\langle \Gamma, \Delta \rangle$ belongs to $\vdash_{\mathcal{L}}$, $\langle \Gamma \cup \Sigma_1, \Delta \cup \Sigma_2 \rangle$ necessarily belongs to $\vdash_{\mathcal{L}}$, for if every token supporting all members of Γ supports at least one member of Δ, then every token supporting all members of the superset $\Gamma \cup \Sigma_1$ of Γ supports at least one member of the superset $\Delta \cup \Sigma_2$ of Δ. This means that the set $\vdash_{\mathcal{L}}$ of sequents must satisfy Weakening. Identity and Global Cut are required for similar reasons. [3]

The local logics involved in the structure in Figure 1 are the following:

- $\mathcal{L}_S = \langle \mathbf{S}, \vdash_{\mathcal{L}_S}, N_{\mathcal{L}_S} \rangle$ (placed in upper left)
- $\mathcal{L}_C = \langle \mathbf{C}, \vdash_{\mathcal{L}_C}, N_{\mathcal{L}_C} \rangle$ (upper center)
- $\mathcal{L}_T = \langle \mathbf{T}, \vdash_{\mathcal{L}_T}, N_{\mathcal{L}_T} \rangle$ (upper right)

Definition 3 (Infomorphism). *Given classifications* $\mathbf{A}$ *and* $\mathbf{B}$*, an* infomorphism $f : \mathbf{A} \rightleftarrows \mathbf{B}$ *from* $\mathbf{A}$ *to* $\mathbf{B}$ *is a pair of functions* $f = \langle f\hat{}, f\check{} \rangle$ *such that:*

1. $f\hat{} : typ(\mathbf{A}) \rightarrow typ(\mathbf{B})$*,*
2. $f\check{} : tok(\mathbf{B}) \rightarrow tok(\mathbf{A})$*, and*
3. $f\check{}(b) \models_{\mathbf{A}} \alpha$ *iff* $b \models_{\mathbf{B}} f\hat{}(\alpha)$ *for each token* $b \in tok(\mathbf{B})$ *and each type* $\alpha \in typ(\mathbf{A})$*.*

Thus, the structure in Figure 1 involves two infomorphisms, $f_S : \mathbf{S} \rightleftarrows \mathbf{C}$ and $f_T : \mathbf{T} \rightleftarrows \mathbf{C}$. One, f_S, consists of a function $f_S\hat{}$ from typ($\mathbf{S}$) to typ($\mathbf{C}$) and a

[1] $\langle \Sigma_1, \Sigma_2 \rangle$ is a partition of Σ iff $\Sigma_1 \cup \Sigma_2 = \Sigma$ and $\Sigma_1 \cap \Sigma_2 = \emptyset$. Note that this definition allows Σ_1 or Σ_2 in a partition $\langle \Sigma_1, \Sigma_2 \rangle$ to an empty set, unlike the definition of partition adopted in certain contexts.

[2] Just because you specify a subset $N_{\mathcal{L}}$ of tok($\mathbf{A}$), it does not mean that you have specified the reason why some members of tok($\mathbf{A}$) are in $N_{\mathcal{L}}$ while others out. We will come back to this issue later.

[3] In fact, it has been shown that the satisfaction of these closure conditions is also a *sufficient* condition for a set of sequents to be the set of the constraints on a classification. See Section 9.5 of [1] for the more precise formulation of this idea.

function $f_S{}^{\vee}$ from tok(**C**) to tok(**S**), and the other, f_T, consists of a function $f_T{}^{\wedge}$ from typ(**T**) to typ(**C**) and a function $f_T{}^{\vee}$ from tok(**C**) to tok(**T**). The symbol $\rightleftarrows$ suggests the reversed directions of the two functions involved in an infomorphism. Since the two infomorphisms are both connected to the classification **C**, Barwise and Seligman call it the *core* of this structure.

Now, a representation system is just a pair of infomorphisms with a common core, coupled with a local logic on each of the three classifications involved.

Definition 4 (Representation System). *A representation system $\mathcal{R}$ is a quintuple $\langle f_S : \mathbf{S} \rightleftarrows \mathbf{C}, f_T : \mathbf{T} \rightleftarrows \mathbf{C}, \mathcal{L}_S, \mathcal{L}_C, \mathcal{L}_T \rangle$ where f_S and f_T are infomorphisms and $\mathcal{L}_S$, $\mathcal{L}_C$, and $\mathcal{L}_T$ are local logics on the classifications* **S**, **C**, *and* **T**, *respectively.*

3 Easy Part: The Source and Target Logics

Now that we have laid out a mathematical structure, we start our explication of how it is supposed to capture something in real world. Barwise and Seligman intend an individual representation system $\mathcal{R} = \langle f_S : \mathbf{S} \rightleftarrows \mathbf{C}, f_T : \mathbf{T} \rightleftarrows \mathbf{C}, \mathcal{L}_S, \mathcal{L}_C, \mathcal{L}_T \rangle$ to capture a *practice* of producing representations of a particular kind. Such practices include that of producing maps, drawing diagrams, painting pictures, writing sentences, and uttering sentences. How are the individual components of the structure $\mathcal{R}$ fitted to the components of such a practice? We start with the components of $\mathcal{R}$ that are relatively easy to interpret.

3.1 Source

The classification $\mathbf{S} = \langle \mathrm{tok}(\mathbf{S}), \mathrm{typ}(\mathbf{S}), \models_{\mathbf{S}} \rangle$ depicted in the left side of Figure 1 is called the *source* of the representation system, and the members of tok(**S**) are called *representations*. Here, a representation $a \in \mathrm{tok}(\mathbf{S})$ is intended to be such things as an individual diagram drawn on a particular sheet of paper, a map printed on a particular page of a brochure, and a sentence displayed on a particular computer display. As a token, a representation is distinguished from its appearance. So, when one draws exactly the same arrangement of symbols on two different occasions, the result is two different diagram tokens (representations) although they have exactly the same appearance. Similarly, two prints of *Downtown Chicago Map* published by the same map publisher are different representations—map tokens—even though they usually have exactly the same arrangement of symbols and colors.

In contrast, typ(**S**) consist of syntactic properties that classify the representations in tok(**S**) according to what symbols appear in what arrangements. Thus, if tok(**S**) consists of the individual map prints of *Downtown Chicago Map*, typ(**S**) may contain syntactic properties such as the following:

(σ_1) There is a unique street line labeled "E Ontario," drawn left-right.

(σ_2) There are unique street lines labeled "N Rush" and "N Saint Claire," drawn up-down.

(σ_3) A hotel symbol is at the upper-left corner of the intersection of street lines labeled "N Rush" and "E Ontario."

(σ_4) A hotel symbol is at the lower-right corner of the intersection of street lines labeled "N Saint Claire" and "E Ontario."

These are types, or properties, in the sense that they can hold of many different map tokens. In fact, they hold of all prints of *Downtown Chicago Map*, with the exception of defective or degenerated copies (changed colors, unprinted symbols, spilled coffee, etc.) These types hold of even *revised* maps of downtown Chicago, as long as they have street lines with appropriate labeling, have a certain line straight and certain lines in parallel, and have hotel symbols in certain positions. In this way, given the sets $\mathrm{tok}(\mathbf{S})$ and $\mathrm{typ}(\mathbf{S})$, the relation $\models_{\mathbf{S}}$ is determined by which tokens support which types in reality.

3.2 Target

The classification $\mathbf{T} = \langle \mathrm{tok}(\mathbf{T}), \mathrm{typ}(\mathbf{T}), \models_{\mathbf{T}} \rangle$ depicted in the right side of Figure 1 is called the *target* of the system. Intuitively, a member b of $\mathrm{tok}(\mathbf{T})$ is something that is actually represented in a representational practice, so in the case of the mapmaking practice for *Downtown Chicago Map*, it is a particular region in Chicago in some period of time. If the company produces revised maps, the same region in a subsequent time period gets represented in this mapmaking practice, so it makes another member of $\mathrm{tok}(\mathbf{T})$.

Naturally, $\mathrm{typ}(\mathbf{T})$ consists of types, or properties, that classify these represented objects. In our case of the mapmaking practice, the set consist of types that classify the region of Chicago in different periods of time. It may contain:

(θ_1) There is a unique streets named "E Ontario," running east-west.

(θ_2) There are unique streets named "N Rush" and "N Saint Claire," running north-south.

(θ_3) A hotel building is at the north-west corner of the crossing of streets named "North Rush" and "East Ontario."

(θ_4) A hotel building is at the south-east corner of the crossing of streets named "North Saint Claire" and "East Ontario."

Note the difference from the types σ_1–σ_4. The types θ_1–θ_4 refer to possible arrangements of streets and buildings on a region in Chicago, rather than possible arrangements of lines and symbols on a sheet of paper.

3.3 Local Logic on the Source

The local logic $\mathcal{L}_S$ shown in the upper left part of Figure 1 is designed to capture a system of constraints governing the source classification $\mathbf{S}$. It is the triple $\langle \mathbf{S}, \vdash_{\mathcal{L}_S}, N_{\mathcal{L}_S} \rangle$, and as with every local logic, it does its job by having the second coordinate $\vdash_{\mathcal{L}_S}$ specify a set of sequents that every normal token in $\mathrm{tok}(\mathbf{S})$

satisfies. In our case of mapmaking practice, an example of such a sequent can be $\langle\{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}, \{\sigma_5\}\rangle$, where σ_1–σ_4 are as above and σ_5 is:

(σ_5) There are at least two hotel symbols on a street line labeled "E Ontario."

This particular constraint $\{\sigma_1, \sigma_2, \sigma_3, \sigma_4\} \vdash_{\mathcal{L}_S} \sigma_5$ is due to a geometrical constraint governing lines and symbols on a plane. The set $\vdash_{\mathcal{L}_S}$ may also contain physical constraints on the coloring of symbols. In addition to these natural constraints, the set typically contains constraints due to the *syntactic stipulations* adopted in the mapmaking practice, regulating such things as what types of building symbols can appear, how labels are placed on them, and what varieties of colors can color regions of a map.

3.4 Local Logic on the Target

The local logic $\mathcal{L}_T$ shown in the upper right part of Figure 1 captures a system of constraints governing the target classification $\mathbf{T}$. It is the triple $\langle \mathbf{T}, \vdash_{\mathcal{L}_T}, N_{\mathcal{L}_T}\rangle$, and it works just as the local logic $\mathcal{L}_S$ works on the source classification $\mathbf{S}$. Thus, in our case of mapmaking practice, $\vdash_{\mathcal{L}_T}$ contains such sequents as $\langle\{\theta_1, \theta_2, \theta_3, \theta_4\}, \{\theta_5\}\rangle$, where σ_1–σ_4 are as above and σ_5 is:

(θ_5) There are at least two hotel buildings on a street named "East Ontario."

The correspondence between this constraint $\{\theta_1, \theta_2, \theta_3, \theta_4\} \vdash_{\mathcal{L}_S} \theta_5$ on the target classification and the constraint $\{\sigma_1, \sigma_2, \sigma_3, \sigma_4\} \vdash_{\mathcal{L}_S} \sigma_5$ on the source classification is not an accident. Both constraints are based on the same topological law that regulate buildings and streets on a geographical region, as well symbols and lines on a map.

4 Difficult Part: The Core Logic

So far, we have been concerned with the components of the B&S model that are depicted on the left and the right side of Figure 1, and their interpretation was relatively straightforward. What about the components in the middle? What is the center classification $\mathbf{C}$ supposed to model? What role do the two infomorphisms $f_S = \langle f_S\hat{}, f_S\check{}\rangle$ and $f_T = \langle f_T\hat{}, f_T\check{}\rangle$ play by having $\mathbf{C}$ as its core?

These medial components are there to define what Barwise and Seligman [1] call the *representation relation* and the *indication relation* supported by the representation system.

Definition 5 (Representation and Indication). *Let* $\mathcal{R} = \langle f_S : \mathbf{S} \rightleftarrows \mathbf{C}, f_T : \mathbf{T} \rightleftarrows \mathbf{C}, \mathcal{L}_S, \mathcal{L}_C, \mathcal{L}_T\rangle$ *be a representation system.*

1. *A token a in tok*$(\mathbf{S})$ represents *a token b in tok*$(\mathbf{T})$*, written $a \rightsquigarrow_{\mathcal{R}} b$, if there is a token c in tok*$(\mathbf{C})$ *such that $f_S\check{}(c) = a$ and $f_T\check{}(c) = b$.*
2. *A type α in typ*$(\mathbf{S})$ indicates *a type β in typ*$(\mathbf{T})$*, written $\alpha \Rightarrow_{\mathcal{R}} \beta$, if $f_S\hat{}(\alpha) \vdash_{\mathcal{L}_C} f_T\hat{}(\beta)$.*

In this section, we first explicate the two relations $\rightsquigarrow_{\mathcal{R}}$ and $\Rightarrow_{\mathcal{R}}$ defined above, to explain the framework of "two-tier" semantics central to the B&S model.

4.1 Representation Relation

Intuitively, Clause 1 of Definition 5 says that a token in tok(**S**) represents a token in tok(**T**) if they are mediated by a token c in tok(**C**). Barwise & Seligman [1] characterize this c as "the particular spatial-temporal process whereby the representation comes to represent what it does" (p. 236). When a is a map, c is also characterized a "causal link" between a map and what it is a map of (p. 237). In such a case, the classification **C** is said to model the "actual practice" of mapmaking (p. 236), so it is natural to interpret this causal link c as an *individual act* that belongs to this practice of mapmaking.

Thus, in our case of mapmaking practice for downtown Chicago, a token c_1 in tok(**C**) can be an act, conducted mainly by the map publisher, that consists of the sub-acts of assembling relevant information about a particular region in Chicago in a certain period of time, editing the information in the form expressible in a map, and printing it on a particular sheet of paper. When another map is printed, this new printing act is combined with the first two sub-acts of c_1 to make another token c_2 in tok(**C**).

To see this situation more clearly, let e_1 and e_2 be the sub-acts of information collection and information editing, respectively, and p_1 and p_2 be the first and the second printing acts mentioned above. Then, the representational act c_1 is the sequence $e_1 \circ e_2 \circ p_1$ of sub-acts, while the representational act c_2 is the sequence $e_1 \circ e_2 \circ p_2$. Generally, every act p_i of printing a new map based on the information collected and edited in e_1 and e_2 gives rise to a new representational act $c_i = e_1 \circ e_2 \circ p_i$, a new token in tok(**C**).

Conceived in this way, every individual mapmaking act c has a unique map as its product, and a unique region of Chicago in a particular period of time as the object about which information is assembled and edited. We call the former the *representing object* and the latter the *represented object* of the act c.[4]

The function $f_S\check{}$ can be interpreted as the assignment of a unique object to the role of representing object in every mapmaking act in tok(**C**). Similarly the function $f_T\check{}$ can be interpreted as the assignment of a unique object to the role of represented object in every such mapmaking act.

Combined, the mapmaking act c connects the particular map $f_S\check{}(c)$ to the particular region $f_T\check{}(c)$ in the particular period of time. This is the situation described by Clause 1 of Definition 5. It is a representational act that connects a token in tok(**S**) to a token in tok(**T**), making the former stand in the representation relation $\leadsto_{\mathcal{R}}$ to the latter.

[4] When considering the production of representations via acts, the source of the information used in the act is a token in the target classification, and the result of the act is a token in the source classification. It may appear, then, that they have misnamed our classifications. However the names that we have chosen derive from the more common situation where a diagram is the source of information about a target – utilization, rather than production, of the diagram.

4.2 Indication Relation

The notion of role that we have just introduced to explain the functions $f_S\check{}$ and $f_T\check{}$ also helps us to interpret the other functions $f_S\hat{}(c)$ and $f_T\hat{}(c)$, which are depicted in the upper middle part of Figure 1. Given a type α in typ(**S**), we can think of $f_S\hat{}(\alpha)$ as the type that classifies an act in tok(**C**) on the basis of the property of the object playing the role of represented object in it. For example, if a mapmaking act c has produced a map in which there is a unique street labeled "E Ontario," we can classify c as being of the following type:

(ω_1) The object playing the role of representing object is such that there is a unique street line labeled "E Ontario."

Earlier, we considered the following type as a member of typ(**S**):

(σ_1) There is a unique street line labeled "E Ontario."

The types ω_1 and θ_1 are different, but they are closely related in the way the following equivalence holds:

(1) A mapmaking act c is of type ω_1 if and only if the object playing the role of the representing object in c is of type σ_1.

Thus, ω_1 is the property that classifies a mapmaking act c according to whether the object playing the role of representing object in c is of type σ_1.

Generally, for every type σ in typ(**S**), there is a unique type ω in typ(**C**) that classifies a member of tok(**C**) according to whether the object playing the role of representing object in it is of type σ_i. The model uses the function $f_S\hat{}(c)$ to capture this functional relation from typ(**S**) to typ(**C**). In the case of the types ω_1 and σ_1 above, $f_S\hat{}$ assigns ω_1 to σ_1. So, we can paraphrase (1) as:

(2) A mapmaking act c is of type $f_S\hat{}(\sigma_1)$ if and only if the object playing the role of representing object in c is of type σ_1.

Recalling that the object playing the role of the representing object in c is assigned by $f_S\check{}$ to c, this amounts to saying:

(3) A mapmaking act c is of type $f_S\hat{}(\sigma_1)$ if and only if $f_S\check{}(c)$ is of type σ_1.

Here, we see an instance of the equivalence condition stated in the definition of infomorphism (Definition 3). The condition lets us generalize (3) to every token c in tok(**C**) and every type σ in typ(**S**). This way, the infomorphism $f = \langle f_S\hat{}, f_S\check{}\rangle$ captures the partial type-equivalence between an act c and the object that plays the role of representing object in c.

A similar consideration applies to the infomorphism $f = \langle f_T\hat{}, f_T\check{}\rangle$: it captures the partial type-equivalence between an act c and the object that plays the role of represented object in c. For example, consider the type θ_1 in typ(**T**) and the type ω_2 in typ(**C**):

(θ_1) There is a unique street named "E Ontario."

(ω_2) The object playing the role of represented object is such that there is a unique street named "E Ontario."

Here, $f_T\hat{}$ assigns ω_2 to θ_1, so the following equivalence holds:

(4) A mapmaking act c is of type $f_T\hat{}(\theta_1)$ if and only if $f_T\check{}(c)$ is of type θ_1.

Again, the equivalence condition in Definition 3 lets us generalize (4) to every token in c in tok($\mathbf{C}$) and every type θ in typ($\mathbf{T}$).

Now, by the definitions of $f_S\hat{}$ and $f_T\hat{}$, both the image $f_S\hat{}(\text{typ}(\mathbf{S}))$ and the image $f_T\hat{}(\text{typ}(\mathbf{T}))$ are subsets of typ($\mathbf{C}$). The B&S model uses this fact to define the indication relation from types in typ($\mathbf{S}$) to types in typ($\mathbf{T}$). Take the type σ_1 in typ($\mathbf{S}$) and the type θ_1 in typ($\mathbf{T}$) for example. Intuitively, if there is a unique street line labeled "E Ontario" drawn right-left in a downtown Chicago map, it indicates that there is a unique street named "E Ontario" running east-west in the mapped region. That is, the type σ_1 indicates the type θ_1 in this mapmaking practice. Barwise and Seligman models this indication relation as a constraint in the local logic $\mathcal{L}_C$. For them, σ_1 indicates θ_1 just in case there holds the constraint $f_S\hat{}(\sigma_1) \vdash_{\mathcal{L}_C} f_T\hat{}(\theta_1)$.

Remember that $\vdash_{\mathcal{L}_C}$ lists the constraints governing the classification $\mathbf{C}$ of a set of representational acts. In particular, the constraint $f_S\hat{}(\sigma_1) \vdash_{\mathcal{L}_C} f_T\hat{}(\theta_1)$ states the following:

(5) A representational act in tok($\mathbf{C}$) is such that the representing object (a map, in the present example) is of the type σ_1 only if the represented object (a city region) is of the type θ_1.

This makes it clear that the constraint $f_S\hat{}(\sigma_1) \vdash_{\mathcal{L}_C} f_T\hat{}(\theta_1)$ is something maintained by the effort of people who are involved in the mapmaking acts in tok($\mathbf{C}$). The constraint is essentially *arbitrary* in its origin, but once people start conforming to it and believe that everybody conforms to it, it satisfies significant mutual benefit for them to keep conforming to it. It thus becomes a "self-perpetuating" constraint over the representational acts of a group of people. Lewis [9] has developed a general theory of how such a constraint becomes a stabilized character of human conducts.

The condition $f_S\hat{}(\sigma_1) \vdash_{\mathcal{L}_C} f_T\hat{}(\theta_1)$ is the way Barwise and Seligman capture one of such constraints. Clause 2 of Definition 5 is then a generalization of this strategy of capturing a *semantic constraint* stabilized in a representational practice: it characterizes the indication relation $\Rightarrow_{\mathcal{R}}$ as the relation that holds between a type α in typ($\mathbf{S}$) and a type β in typ($\mathbf{T}$) whenever the constraint of the form $f_S\hat{}(\alpha) \vdash_{\mathcal{L}_C} f_T\hat{}(\beta)$ holds.

Note: Other Constraints on Representational Acts. Typically, our representational acts are constrained not only by these semantic constraints, but also by *syntactic stipulations*, concerning what arrangements of symbols and colors are allowed in representing objects. These stipulations combined with natural (geometrical, topological, and physical) constraints to produce a larger set of constraints on the arrangements of symbols and colors. The B&S model captures their effects as constraints in the local logic $\mathcal{L}_S$ on the source classification, not as constraints in the local logic $\mathcal{L}_C$ on the core classification.

Moreover, our representational acts are typically constrained by *target restrictions*, namely, restrictions on the choice of objects to be represented. For example, only a region of Chicago in some period is chosen as the represented object in the representational practice for *Downtown Chicago Map*. In fact, this is an assumption on which the syntactic stipulations and semantic conventions of this mapmaking practice are stabilized. Totally different stipulations and conventions would be adopted if a region of the Rocky Mountains were the main target of the representational practice. The kind of objects that come in the set tok($\mathbf{T}$) is thus restricted due to target restrictions on our representational acts, and for this reason, a substantial set of constraints hold on the classification $\mathbf{T}$ and get captured in the local logic $\mathcal{L}_T$ on the target classification. The constraint $\{\theta_1, \theta_2, \theta_3, \theta_4\} \vdash_{\mathcal{L}_S} \theta_5$ cited in Section 3.4 is an example of such constraints.

4.3 Two-Tier Semantics

This pair of relations $\rightsquigarrow_{\mathcal{R}}$ and $\Rightarrow_{\mathcal{R}}$ that we have just explained lets us characterize the informational relation between a representation and the represented object in a natural way.

Definition 6 (Representing As). *Let* $\mathcal{R} = \langle f_S : \mathbf{S} \rightleftarrows \mathbf{C}, f_T : \mathbf{T} \rightleftarrows \mathbf{C}, \mathcal{L}_S, \mathcal{L}_C, \mathcal{L}_T \rangle$ *be a representation system. Given a token* a *in tok*($\mathbf{S}$)*, a token* b *in tok*($\mathbf{T}$)*, and a type* β *in typ*($\mathbf{T}$)*,* a represents b as being of type β *if*

1. $a \rightsquigarrow_{\mathcal{R}} b$ *and*
2. *there is a type* α *in typ*($\mathbf{S}$) *such that*
   - $a \models_{\mathbf{S}} \alpha$*, and*
   - $\alpha \Rightarrow_{\mathcal{R}} \beta$.

For example, recall that a map publisher takes an act whose representing object is a particular complete print a of *Downtown Chicago Map* and whose represented object is a particular region b of Chicago in a particular period of time (i.e., $a \rightsquigarrow_{\mathcal{R}} b$), that there is a unique street line labeled "E Ontario" drawn right-left in this particular print a (i.e., $a \models_{\mathbf{S}} \sigma_1$), and that it is a constraint on this mapmaking practice that one tries to produce a map with a unique road line labeled "E Ontario" drawn right-left only if the represented object has a unique street named "E Ontario" running east-west (i.e., ${f_S}\hat{}(\sigma_1) \vdash_{\mathcal{L}_C} {f_T}\hat{}(\theta_1)$ and hence $\sigma_1 \Rightarrow_{\mathcal{R}} \theta_1$). Under these conditions, the particular print a of *Downtown Chicago Map* is said to *represent* the region b of Chicago in the particular period of time *as* having a unique road named "E Ontario."

The semantic theory outlined by Definitions 5 and 6 is "two-tier" in that it is formulated with reference to two relations $\rightsquigarrow_{\mathcal{R}}$ and $\Rightarrow_{\mathcal{R}}$. Historically, we can see it as a formal realization of some key ideas of situation semantics [10,11]: (1) It takes the primary carrier of meaning to be a particular object in the world (the token a in Definition 5 and 6) rather than a representation type, (2) it takes a representation as carrying meaning about some particular object in the world (the token b in Definition 5 and 6) rather than an entire (possible) world, and (3) it takes meaning as a special case of information-carrying regularities holding in the environment (Clause 2, Definition 5).

5 Notion of Representational Practice

Our explanations so far have clarified what kinds of objects constitute each of these sets tok($\mathbf{C}$), tok($\mathbf{S}$), and tok($\mathbf{T}$). The set tok($\mathbf{C}$) consists of representational acts that collect information about unique objects to produce unique representations, while tok($\mathbf{S}$) representations produced by the representational acts in tok($\mathbf{C}$) and tok($\mathbf{T}$) represented objects in the representational acts in tok($\mathbf{C}$). As this description makes clear, tok($\mathbf{S}$) and tok($\mathbf{T}$) are defined on the basis of tok($\mathbf{C}$), so they can be defined once tok($\mathbf{C}$) is defined.

The remaining problem is that it is by no means trivial to define tok($\mathbf{C}$). We have seen that Barwise and Seligman conceptualize a representation system $\mathcal{R}$ as a model of a representational *practice*, so that the members of tok($\mathbf{C}$) are individual acts that constitute a representational practice. So the question is what makes an individual act a member of a particular representational practice rather than another. What distinguishes a representational practice from an arbitrarily chosen set of individual acts?

This question is of utmost importance, because what exactly tok($\mathbf{C}$) is determines what constraints hold on the members of tok($\mathbf{C}$), that is, what semantic conventions the relevant representational practice conform to. Furthermore, as tok($\mathbf{S}$) and tok($\mathbf{S}$) are determined on the basis of tok($\mathbf{C}$), the question has ramifications on the set of constraints that the representations in the practice conform to, as well the set of constraints that the objects represented in the practice conform to. All these profoundly affect the effectiveness of the representational practice in question.

Unfortunately, Barwise and Seligman provide no positive clue about this issue. However, it is clear that the satisfaction of all constraints listed in $\vdash_{\mathcal{L}_C}$ *cannot* be the defining character of the set tok($\mathbf{C}$). The definition of a local logic requires all constraints to be satisfied by all members of $N_{\mathcal{L}_C}$, but it does not require them to be satisfied by all members of tok($\mathbf{C}$).

Our proposal is to adopt Millikan's idea of reproduction [7,8] and define tok($\mathbf{C}$) as the class of objects "having been produced from one another or from the same models" [8, p. 20]. A typical example of such a reproductive class is the handshakes occurring in a single culture. Except for a small number of early instances in the history of this culture, individual acts of handshake are not products of somebody's creation, but reproductions of some previous handshakes.

Generally, the members of a reproductive class share certain characters, precisely because they are copies of one another. Thus, handshakes share various physical characters in their movements, and that is because they are reproductions of some previous handshakes or some model handshakes. Following Millikan [7], let us call such a character preserved in a repeated reproduction process a *reproductively established character.* Thanks to humans' ability to copy or repeat previous acts of handshakes, the reproduction process is relatively stable, producing faithful copies over a period of time. However, it does go awry sometimes, resulting in an anomalous handshake that lacks the relevant reproductively established character.

To apply these ideas to characterize the set $\text{tok}(\mathbf{C})$, let $\mathbb{C}$ be the character of "satisfying a set of syntactic stipulations, semantic constraints, and target restrictions." Then we can see $\text{tok}(\mathbf{C})$ as a reproductive class having $\mathbb{C}$ as its reproductively established character. On this conception, each representational act in $\text{tok}(\mathbf{C})$ is a reproduction of previous representational acts in $\text{tok}(\mathbf{C})$, and precisely because it is a reproduction, it tends to share the character $\mathbb{C}$ with other members of $\text{tok}(\mathbf{C})$. Again, the process of reproduction is generally stable, but it can go awry, and when it does, there is no guarantee that the resulting act has the character $\mathbb{C}$, that is, it can violate some of the syntactic stipulations, semantic constraints, and target restrictions. We can then interpret $N_{\mathcal{L}_C}$ as those acts for which the reproduction process goes normally and $\text{tok}(\mathbf{C}) - N_{\mathcal{L}_C}$ as those for which the reproduction process goes awry.

Examples

Example 1: System of Downtown Chicago Maps. In the case of the mapmaking practice for downtown Chicago, some people collect information about a region in Chicago in a particular period of time, some designer for the map publisher uses that information to design a map, and the technician prints the first print of the map. This sequence of acts, which we have characterized as $c_1 = e_1 \circ e_2 \circ p_1$ before, is the first token in $\text{tok}(\mathbf{C})$. This act satisfies a set of syntactic stipulations, semantic constraints, and target restrictions, and we call this character $\mathbb{C}$. When the technician makes another print, this act p_2 is sequenced with the previous acts e_1 and e_2 to make another representational act, $c_2 = e_1 \circ e_2 \circ p_2$, which is a reproduction of the act $c_1 = e_1 \circ e_2 \circ p_1$, and as such shares the character $\mathbb{C}$ with it. Another printing act then gives rise to another representational act $c_3 = e_1 \circ e_2 \circ p_3$, and so on until the final reproductive act, say, $c_{2875} = e_1 \circ e_2 \circ p_{2875}$. This way, multiple representational acts are reproduced, preserving the character $\mathbb{C}$. The class $\text{tok}(\mathbf{C})$ of these acts is a reproductive class, and this makes up a mapmaking practice for downtown Chicago.

What happens if the map publisher decides to update the maps they publish, to reflect the recent change in the mapped region? Then a new sequence of an information-collecting act, a design act, and a map-printing act takes place, making up a representational act, say c_{2876}. If the reproductive process is of the kind that normally preserves $\mathbb{C}$, c_{2876} belongs to the same reproductive class as $c_1, c_2, \ldots, c_{2875}$ do. Then all subsequent acts reproducing c_{2876} are all members of this class $\text{tok}(\mathbf{C})$, extending the map publisher's mapmaking practice.

Example 2: System of Scheduling Tables. Let us consider a case where representations are used in more private settings. If the manager of a shop regularly hears from the part-time workers to decide on who will work on what day of the next week, and she puts up a table of the next week's work schedule on the door of her office, that will construct a reproductive class of representational acts. The reproductively established character $\mathbb{C}$ is the satisfaction of syntactic stipulations (regulating, for example, what types of symbols can appear in a table cell), semantic constraints (regulating, for example, what types of symbols indicate "on" and what types "off"), and target restriction limiting a representational act only to the weekly work schedules of this particular shop. The manager's act of hearing and table-drawing conducted each week makes an individual representational act, where the represented object is the shop's work schedule during the week about which the manager hears from the workers, and the representing object is the particular table drawn by the manager. The act is reproduced every week, accumulating as the tokens in tok($\mathbf{C}$).

Unlike the case of *Downtown Chicago Map*, the arrangement of symbols in the produced table typically changes act by act, but these acts are still members of the reproductive class they are continuations of the same reproductive process that normally preserves $\mathbb{C}$. For the same reason, a representational act done by the sub-manager can be considered a member of the same reproductive class if he or she has copied the manager's table-drawing acts with respect to $\mathbb{C}$.

What if the sub-manager keeps using the same table format but makes a small change in syntactic and semantic constraints, such as using a check mark rather than a circle to indicate "on"? On the one hand, this act can be considered a member of a new reproductive class, which itself can grow if it is copied by further acts. The new class has a slightly different reproductively established character $\mathbb{C}'$ than the class tok($\mathbf{C}$) of the manager's original representational acts. On the other hand, this act is a partial reproduction of the manager's acts, which preserves most of the constraints on them. Thus, this act, as well as any further acts that copies it, can be considered members of a larger reproductive class that extends tok($\mathbf{C}$). This larger reproductive class can be modeled by a B&S model just as well, and since it still preserves a significant set of constraints, it probably merits a serious logical investigation. This reproductive class makes a higher *taxon* of representational acts, whose lower taxa tok($\mathbf{C}$) and the newly started reproductive class.

6 Conclusion

Thanks to all this philosophical discussion, we now have a better conceptual foundation on which we apply the B&S model. It was not clear from Barwise and Seligman's sketch what exactly their model is a model of—what exactly they conceive as a representation system. Our study reveals that it is, roughly, a reproductive class of individual representational acts that inherits a set of representational rules. Our initial test shows that this concept does a pretty good job in carving out a piece of reality that a B&S model is to capture.

Now, the B&S model features three local logics $\mathcal{L}_C$, $\mathcal{L}_S$, and $\mathcal{L}_T$ as its main ingredients. It lets us characterize various interesting properties of representation systems as conditions on $\mathcal{L}_C$, $\mathcal{L}_S$, and $\mathcal{L}_T$, and investigate the logical consequences of these properties. For example, Barwise and Seligman have shown that the properties such as free ride, over-specificity, and auto-consistency of diagrammatic representations can be captured and studied in this way. We believe that the triple of $\mathcal{L}_C$, $\mathcal{L}_S$, and $\mathcal{L}_T$ embeds many more interesting properties, but speculation aside, that work of defining such properties and investigating their consequences is mathematical in nature, since it is exclusively concerned with the model itself and the model in this case is a mathematical structure.

This entails that whatever result it obtains is fundamentally hypothetical—it states that a representation system has such and such properties assuming it has such and such $\mathcal{L}_C$, $\mathcal{L}_S$, and $\mathcal{L}_T$ as its core logic, source logic, and target logic. We do not have to be concerned with which system in the world has such a combination of logics, and for that matter, if there is such a system at all. In fact, it was not possible to be concerned with these matters, since we did not know exactly what a representation system is that a B&S model is a model of.

The philosophical explication in the present paper changes this situation. Now that we know what piece of reality a B&S model is to capture, the specifications of $\mathcal{L}_C$, $\mathcal{L}_S$, and $\mathcal{L}_T$ in a B&S model can be put in empirical test. We have an independent grasp of what the model is about, so we can put the model and the modeled object side by side and evaluate their fit. And when the fit is good, we can say, categorically, that *this* representation system has such and such properties, with a clear understanding of what "this system" refers to. As examples in Section 5 suggest, reproductive classes of representational acts inheriting a set of representational rules are abundant in both private and public media. The B&S model now seems ready for use in the exploration of their logical properties.

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